Linear Programming - ACTIVITIES

ACTIVITY ONE

Learning Objectives

LO1. Students to construct simple objective functions and constraints.

LO2. Students to identify simple feasible regions and optimal points.

Students are put into small groups and given large and laminated graph paper.

The axes are unlabelled and unscaled and the lamination allows students to ‘write and wipe’.

Students are given thin cardboard strips to represent linear constraints.

TASK ONE

In your small groups read the following passage and answer the questions which follow

“MFG is small car manufacturer producing two models of handmade cars. These cars are prestige vehicles and command a very high price. The two models are well known throughout the specialist car market as X and Y. Both car X and car Y are very profitable. Car X produces £2,000 of profit for each vehicle produced whilst Car Y produces £3,000. the firm is motivated purely by profit.

The company cannot produce as many cars as it would like. It cannot produce more than 15 cars in a year because they all have to be hand finished. For operational reasons, there are also limitations on the combinations of cars X and Y which can be manufactured in a given year. It is not possible given the current technology for more than 3 model Y cars to be produced for each model X car. This can create a headache for MFG.”

(a) Write in words MFG’s objective function

(b) In your group, translate the objective function into a simple expression using X to show the number of model X and Y to denote the number of model Y vehicles that can be manufactured in a year.

(c) How many constraints are there to this problem? Work out what the expressions are for these constraints.

(d) Using your laminated paper and strips, construct the linear program. How many of each should be produced?

ACTIVITY TWO

Learning Objectives

LO1. Students to consolidate knowledge and understanding of linear programming using a graphical method.

LO2. Students to be able to independently interpret the results from a graphical linear program and to identify limitations.

Task One

Which statements are TRUE and which are FALSE?

Statement or Assertion	Tick (✓)	
	True	False
A linear programme cannot have more than five constraints.		
Linear programming must always result in an economic variable being maximised.		
The feasible area shows all of the possible combinations of attainable solutions.		
It is possible to have more than one optimal solution.		
An objective function is a qualitative expression summarising a goal or target of an economic agent.		
A constraint is a mathematical representation of a limitation or restriction that an economic agent faces.		

Statement or Assertion	Tick (✓)	
	True	False
A firm is usually assumed to be a profit-maximiser but could have other objectives.		
All economic problems can be represented by linear programmes and solved.		

Task Two

Consider the LP problem below.

The CVH Company of Leeds is a revenue maximising firm. It has chosen to focus on the production and sale of two goods: A and B.

Good A requires 1 hour of skilled labour and 4 kilos of metal. It is a standard product and retails for £40.

Good B is a higher quality version of good A. It needs 2 hours of skilled labour and demands 3 kilos of metal. It sells for a slightly higher price of £50 per unit.

Under current operating conditions, the firm is prepared to hire 40 hours worth of labour each day and it has a long term contract with a supplier who can deliver 120 kilos of metal each day.

Your task is to construct a simple LP and graph to solve.

Task Three (Extension Task)

Look back at the LP you constructed for Task Two. One of the key variables was labour. The case study suggested that a maximum of 40 hours of labour was available each day. Suppose this was comprised of 5 workers who could each work 8 hours each day.

(i) What implicit assumptions have we made about these workers?

(ii) Do you think these assumptions are realistic?

(iii) In what cases could labour be assumed to be entirely identical or homogenous?

Linear Programming - ANSWERS

ACTIVITY ONE

TASK ONE

(a) A descriptive objective function could be, “To maximise profit from the production of vehicle X and Y”

(b) This objective function could be articulated as:

Let P = profit from the production and sale of X and Y cars in a year.

Let X = the number of vehicle X produced and sold in a year.

Let Y = the number of vehicle Y produced and sold in a year.

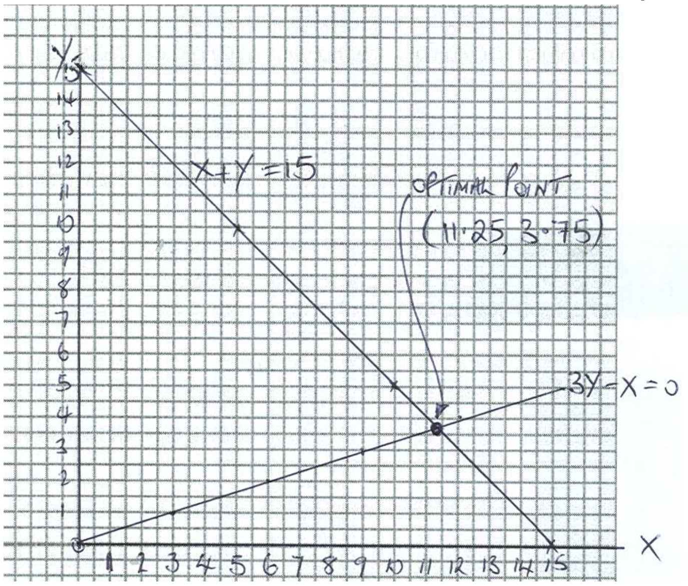
Therefore, P = 2000X + 3000Y

(c) There are two constraints: they cannot produce more than 15 cars in a year and they cannot produce more than 3 lots of Y for every 1 of X.

Constraint 1: $X+Y \leq 15$

Constraint 2: $3Y \leq X$ or $3Y-X \leq 0$

The solution is shown below but in summary we find:



X= 11.25, Y = 3.75, P = £33,750

Students might discuss how meaningful the figure 11.25 is: can 0.25 of car be made and sold?

ACTIVITY TWO

Task One

Statement or Assertion	Tick (✓) True	Tick (✓) False	Reasoning
A linear programme cannot have more than five constraints.		✓	A linear programme can have a large number of constraints.
Linear programming must always result in an economic variable being maximised.		✓	An LP could result in something being minimised e.g. costs.
The feasible area shows all of the possible combinations of attainable solutions.	✓		A feasible area shows all of the possible solutions but usually there is a single point within the feasible area which provides an optimal solution.
It is possible to have more than one optimal solution.	✓		It is possible to have more than one optimal solution e.g. where an objective function and a constraint coincide for a range of values.
An objective function is a qualitative expression summarising a goal or target of an economic agent.		✓	No – it is a mathematical and quantitative expression. Sometimes it can be a quantitative expression of something which is qualitative in nature e.g. a social cost.

Statement or Assertion	Tick (✓)		Reasoning
	True	False	
A constraint is a mathematical representation of a limitation or restriction that an economic agent faces.	✓		A constraint is an expression of a limitation e.g. a household's spending is limited by income.
A firm is usually assumed to be a profit-maximiser but could have other objectives.	✓		Some firms may wish to maximise sales e.g. if senior managers are paid on a commission basis they could be more likely to seek to expand sales rather than profits.
All economic problems can be represented by linear programmes and solved.		✓	In reality, few economic problems can be summarised as simply and neatly as an LP would suggest. They can however provide useful and insightful approximations to how 'the real world' operates.

Task Two

The objective function is to maximise revenue (TR) which is

Max TR = 40A + 50B subject to:

① A + 2B =< 40 hours per hour of skilled workers' time

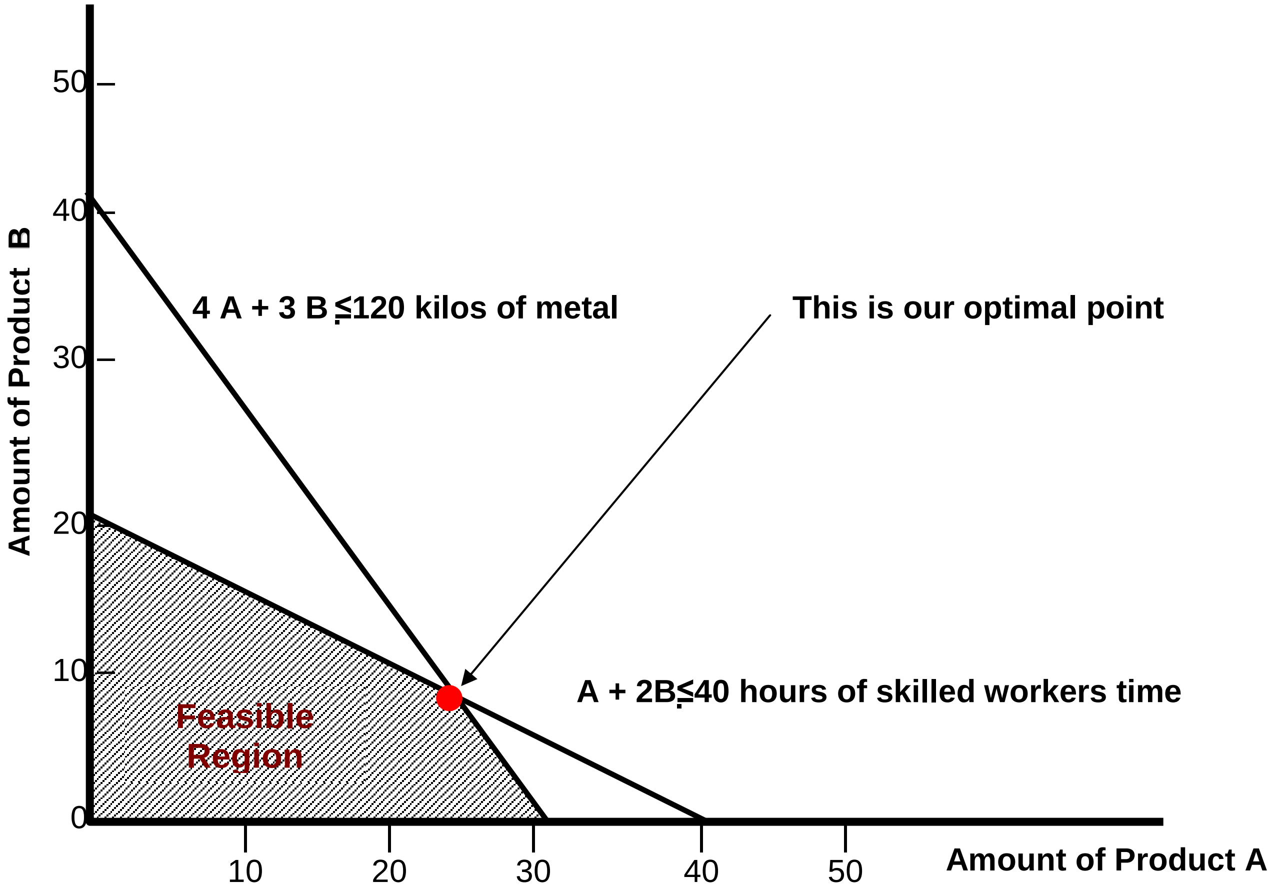
② 4A + 3B =< 120 kilos of metal

③ A and B => 0

The answer is A = 24 and B = 8

Total revenue = (40 * 24) + (50*8) = 960 + 400 = **<u>£1360</u>**

The full graphical solution is shown below:



Task Three (extension)

(i) The LP model presupposes that all of the workers are identical: they have the same set of skills they work equally hard and they produce work at the same rate and of the same identical quality.

(ii) The assumption that all of the workers are identical is clearly a strong assumption. Can we ever really say that two workers are equal in every respect? It is rare that two workers could have their productivity precisely measured and even if this could be done it would be very surprising if the results turned out to be exactly equal. Perhaps the real question is not whether the assumption is realistic but whether the assumption really affects the analysis or our conclusions. There, although the assumption is strong it does not have a material impact on our conclusion or our results.

(iii) Labour is rarely, if ever, completely homogenous. It is more likely to be uniform where simple or low level skills are required or where productivity is measured more in terms of the quantity produced by a worker rather than the quality of what they produce. An example could include manual work e.g. digging a hole in the road. Other examples could include ‘mechanical labour’ e.g. the use of robotics in car factory. In this instance, it could be relatively straightforward to measure the quantity and quality of what is produced but again this is likely to be confined to routine or repetitive manual tasks e.g. multiple spot welding in a plant assembly large commercial aircraft.