Differentiation - ACTIVITIES

ACTIVITY ONE

Learning Objectives

LO1: Students learn how to independently calculate simple derivatives

LO2: Students learn some of the real world applications of differentiation

Task One

The following 5 functions are observed as possible relationships between income (Y) and investment (I) in 5 EU states. In each case you need to calculate the rate of change between investment and income i.e. $\frac{\partial Y}{\partial I}$

State 1: $Y = I^5$

State 2: $Y = I^3$

State 3: $Y = 12I^2$

State 4: $Y = 6$

State 5: $Y = \frac{5}{I}$

Task Two

LO1: Students to learn how to graph functions

LO2: Students learn how to interpret a derived function.

Students can refer to the PowerPoint slides, particularly Slide 7.

To begin, draw out a (non-linear) function, then underneath introduce the axis in which you plan to draw the derived function. Crucial here is an interpretation of what the number on both the axes, and in particular the y axis, actually mean. Students can then trace out what the derived function will look like using the original function as the values of *x* change.

Differentiation - ANSWERS

ACTIVITY ONE

Task One

State 1: $\frac{\partial Y}{\partial I} = 5I^4$

State 2: $\frac{\partial Y}{\partial I} = 3I^2$

State 3: $\frac{\partial Y}{\partial I} = 24I$

State 4: $\frac{\partial Y}{\partial I} = 0$

State 5: $\frac{\partial Y}{\partial I} = \frac{-5}{I^2}$

Task Two

Indeterminate number of answers and outcomes.

Practical Applications - Elasticity and Optimisation - ACTIVITIES

ACTIVITY ONE: The 'Horse Race' Game

Learning objectives

LO1: Students learn how to solve simple problems involving differentiation.

LO2: Students learn how to work independently and in small teams to answer problems concerning 'applied differentiation'.

Understanding and learning mathematics requires practice on the part of students. This understandably can be a boring task to undertake. To make it more interesting an element of competition can often be useful.

The horse race game operates under a quiz format in which students race to answer questions presented to them. The game speeds up somewhat if these are asked as multiple choice questions. This format can be used to look at issues regarding respect to cost, revenue and profit functions which lend themselves to the idea of being broken into stages. For example, a set of questions taken from a standard format used in problem classes breaks neatly into 10 stages.

A horse represents a group of students. The number of horses in the race can obviously be varied with the size of the class and students often enjoy coming up with names for their horses. The only thing to remember is to set enough questions so that one team can complete the race.

Task One

Let P = 20 – 5Q be a demand function

a. how many units will the firm sell if the price is 15?

b. what price should the firm set if it wants to sell 3 units?

c. compute the marginal revenue corresponding to this function

d. calculate price elasticity of demand when price moves from 1 to 3.

e. what is the relationship between the slope of the demand curve and the price elasticity of demand?

f. Calculate the total revenue function for the firm and find its maximum

Task Two

A firms total revenue function is given as follows,

$$TR = 100Q - 2Q^2$$

a. What is the demand function for the firm?

b. Find the marginal revenue for the firm.

c. Does this firm operate in a perfectly competitive industry? Justify this answer.

d. At what points is total revenue maximised?

e. If the government sets a tax equal to tQ, where t = 2, find the new revenue maximizing point?

Task Three

This task has a further Learning Objective

LO3: Students learn how to calculate ths second derivative and use this to solve an economic problem

A firm produces output (Q) using labour (L) and capital (K), according to the following production function.

$$Q = 10KL^{1/2}$$

a. If the firm is using two units of capital and nine units of labour how much output is the firm producing?

b. Assume that capital is fixed in the short run at 2, what is the firm's short run production function?

c. Does the production function satisfy the law of diminishing marginal returns?

Elasticity and Optimisation - ANSWERS

ACTIVITY ONE

Task One

a. Q=1

b. P=5

c. $MR = \frac{\partial TR}{\partial Q} = 20 - 10Q$

d. PeD = $\eta = \frac{\%\Delta Q}{\%\Delta P} = \left(\frac{-2/19 \times 100\%}{+2 \times 100\%} \right) = 0.053$

e. The price elasticity varies as we move along the demand curve. The ped falls as we move from left to right.

f.
$$
\begin{aligned}
&P = 20 - 5Q \\
&TR = 20Q - 5Q^2 \\
&\max TRstQ \\
&\frac{\partial TR}{\partial Q} = 10q - 20 = 0 \\
&\Rightarrow Q = 2 \\
&\Rightarrow TR = 20 \\
&\Rightarrow P = 10
\end{aligned}
$$

Task Two

a. $AR = P = 100 - 2Q$

b. $MR = \frac{\partial TR}{\partial Q} = 100 - 4Q = 100 - 4Q$

c. No because $AR \neq MR$

d. $\mathrm{MR}_{\max}$ when $\mathrm{Q} = 25$

e. Tax of tQ where t =2

$$
\begin{aligned}
&MR = 100 - 4Q - 2Q \\
&6Q = 100 \\
&Q = 16.66
\end{aligned}
$$

Task Three

a. Q=60

b. If k=2 then $Q=20L^{½}$

c. Diminishing marginal returns occur if: $\frac{\partial^2 Q}{\partial L^2} \langle 1$

$$Q = 20L^{\frac{1}{2}}$$
$$\frac{\partial Q}{\partial L} = 10L^{-\frac{1}{2}}$$
$$\frac{\partial^2 Q}{\partial L^2} = -5L^{-\frac{3}{2}}$$

And given that L≥0 then diminishing marginal return must exist.