Solving simultaneous equations using graphs - ACTIVITIES

Learning Outcomes

LO1: Students learn how to solve simultaneous equations

LO2: Students learn how simultaneous equations can help to solve economic problems

TASK ONE

This is a learning activity that could be run in a small group. The group could be divided into three subgroups. Each subgroup would be assigned one of the following three systems of equations in *x* and *y*, and would be asked to present to the whole group their answer to the questions that follow.

System A

$$y = 1 + x \qquad (1)$$
$$y = 3 + x \qquad (2)$$

System B

$$y = 1 + x \qquad (1)$$
$$y = 3 - x \qquad (2)$$

System C

$$y = 1 + x \qquad (1)$$
$$y = 1 + x \qquad (2)$$

For the system of equations that you have been allocated, sketch a graph showing the lines represented by the two equations. In each case, use the graph to determine whether the system has:

(i) A unique solution

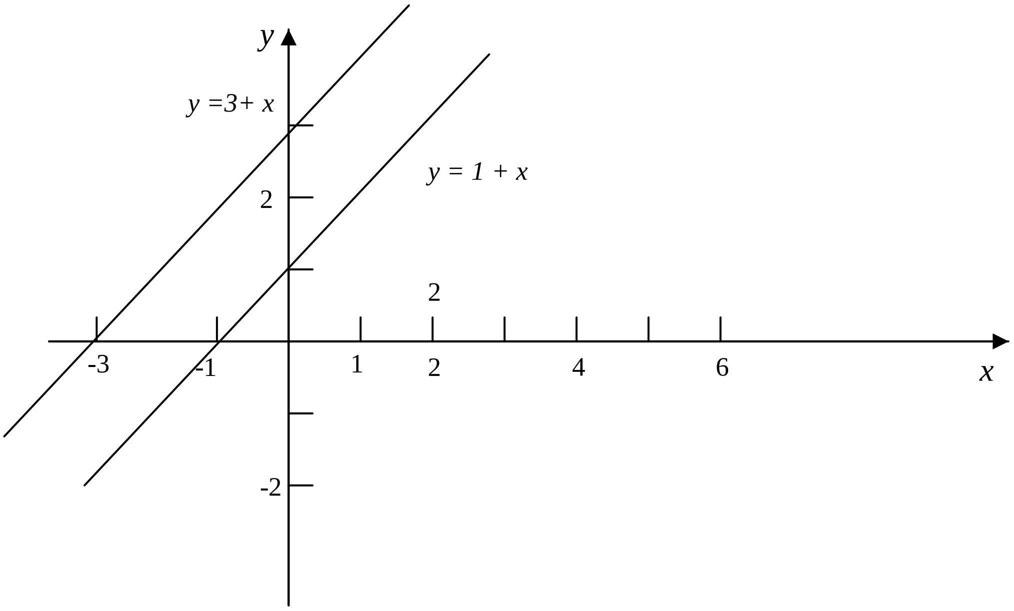
(ii) No solution

(iii) An infinite number of solutions

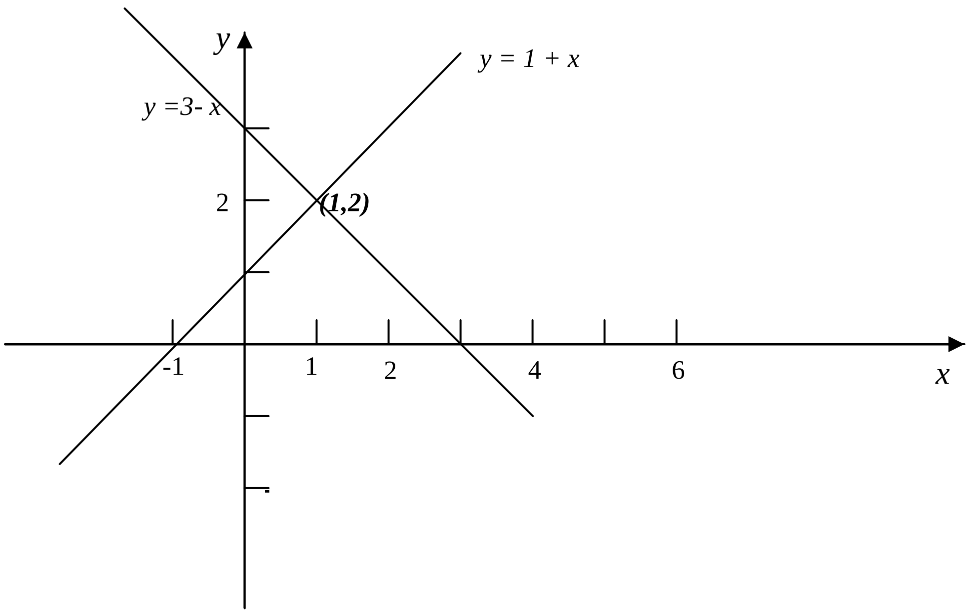
For the system which has a unique solution, solve for *x* and *y*.

Solving simultaneous equations using graphs -ANSWERS

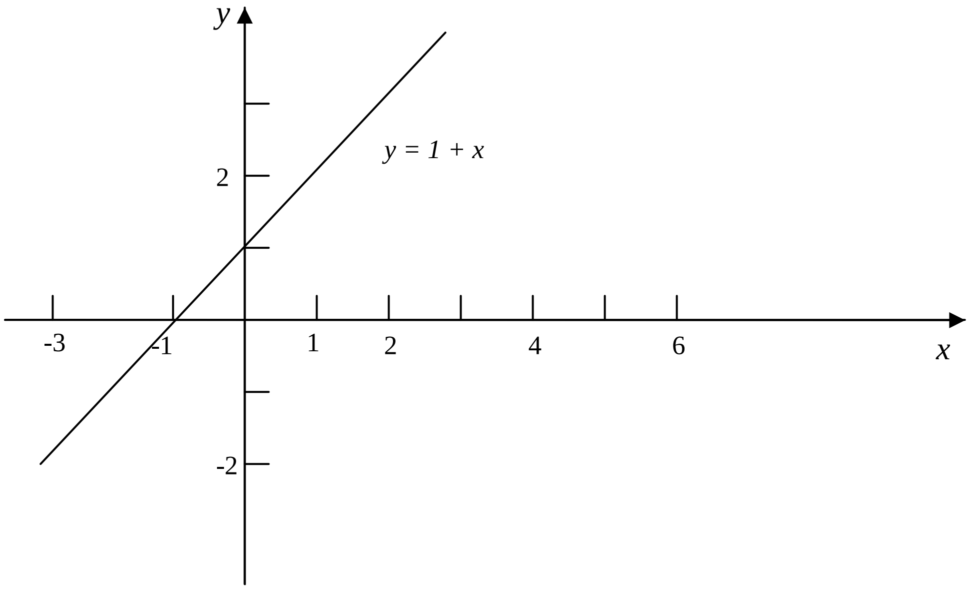
System A



System A has no solutions



System B has a solution at (1,2)



System C has an infinite number of solutions: in effect every point on the line offers a solution because the two equations are identical

Solving simultaneous equations using algebra - ACTIVITIES

ACTIVITY ONE

Learning Objectives

LO1: Students learn how to recognise and specify simultaneous equations

LO2: Students learn how to independently solve simultaneous equations

LO3: Students learn the special case where there are no solutions or an infinite number of solutions

Task One

Solve:

$$3x + 2y = 5 \qquad (1)$$
$$x - 3y = 9 \qquad (2)$$

Task Two

This Activity can be run in a small group, by dividing students into three subgroups with each subgroups being assigned to one of three systems, which they are then asked to solve using algebra, and also to present their solution.

(A)
$$4x + 2y = 2 \qquad (1)$$
$$-5x - y = 2 \qquad (2)$$

(B)
$$2x - 5y = 20 \qquad (1)$$
$$x + 4y = -3 \qquad (2)$$

(C)
$$3x - 2y = -1 \qquad (1)$$
$$x + y = 8 \qquad (2)$$

Use algebra to solve the system of equations that you have been allocated.

Verify that your solution is correct by substituting into the equation that you have not used.

Task Three

As an economist you find two economic relationships between income (y) and consumption (x) as follows:

Observation 1: $y=- -2 + 2x$

Observation 2: $y= 2x$

(a) Graph these two equations and solve. What can you conclude about the type of function relationship and the problem you had trying to solve?

You find two more observations:

Observation 3: $y= -2 + 2x$

Observation 4: $y -4 + 4x$

(b) Graph these two functions and try to solve. What do you notice?

Solving simultaneous equations using algebra - ANSWERS

ACTIVITY ONE

Task One

We first need to multiply every term in equation (2) by 3:

$$3x - 9y = 27 \qquad (3)$$

We are then able to eliminate *x* by subtracting equation (3) from equation (1):

(1)-(3) $\qquad 11y = -22$

$$\therefore \underline{\underline{y = -2}}$$

Then substitute the solution for *y* into (1):

$$3x + 2(-2) = 5$$
$$\therefore 3x - 4 = 5$$
$$\therefore 3x = 9$$
$$\therefore \underline{\underline{x = 3}}$$

Finally, we may check the solution *x* = 3, *y* = -2, using (2).

Task Two

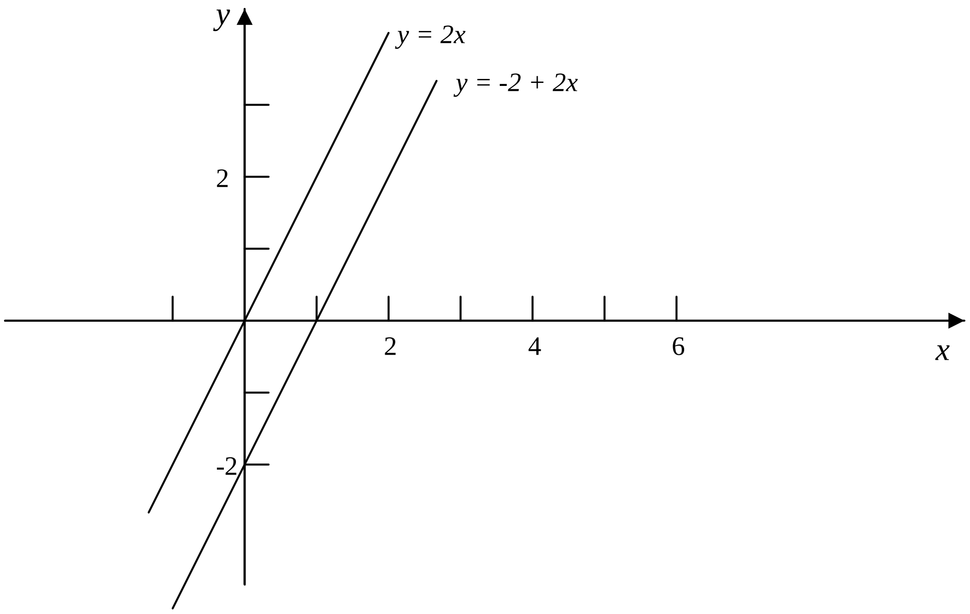
(A) x=-1, y=+3

(B) x=+5, y=-2

(C) x=+3, y=+5

Task Three

(a)



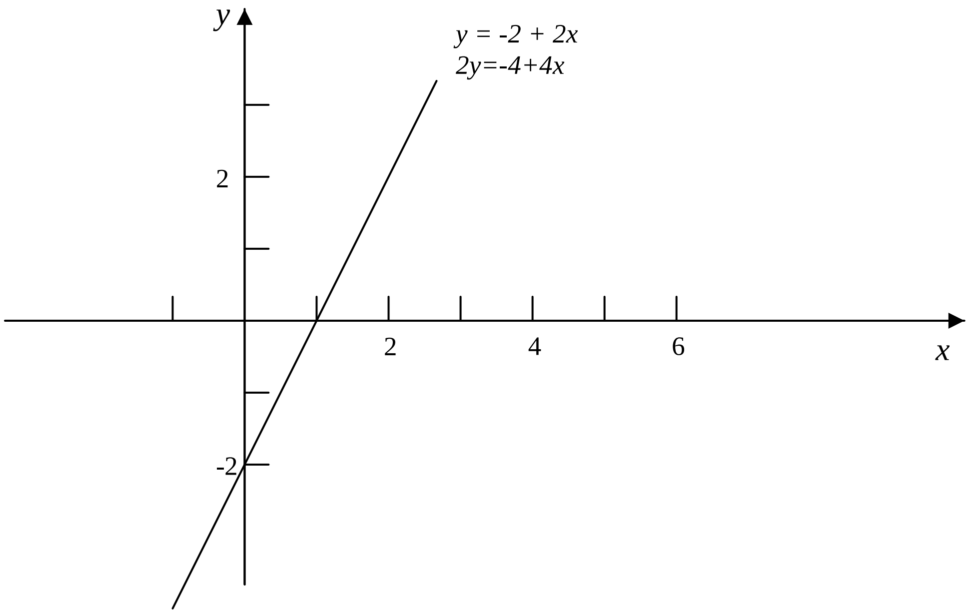
We have a situation in which the two lines run parallel to each other, that is, the two equations have the same slope but different intercepts. It is well known that "parallel lines never meet", so such a pair of equations has no solution.

This problem case is illustrated in Figure 3.2, in which the two equations are:

$$y = -2 + 2x$$
$$y = 2x$$

Because these two lines have the same slope (2), but different intercepts, they are parallel, implying that there is no solution to the pair of equations represented by the two lines.

(b)



Here the two lines coincide. This would happen if the two equations had the same slope and the same intercept, i.e. they are the same equation. In this situation, every point on the line is an intersection of the two lines, and therefore every point on the line represents a solution to the pair of equations. In this situation, we say that the pair of equations has an <u>infinite number of solutions</u>.

Economic Applications - Supply and Demand: Equilibria, Consumer and Producer Surplus, Revenue and Taxation - ACTIVITIES

ACTIVITY ONE

Learning Objectives

LO1: Students learn how to solve simultaneous equations

LO2: Students learn how to apply their knowledge of simultaneous equations to micro- and macroeconomic problems

TASK ONE (Supply and Demand)

The Demand and Supply equations for a particular good are:

D: $Q = 100 - \frac{1}{2}P$ S: $Q = -100 + 2P$

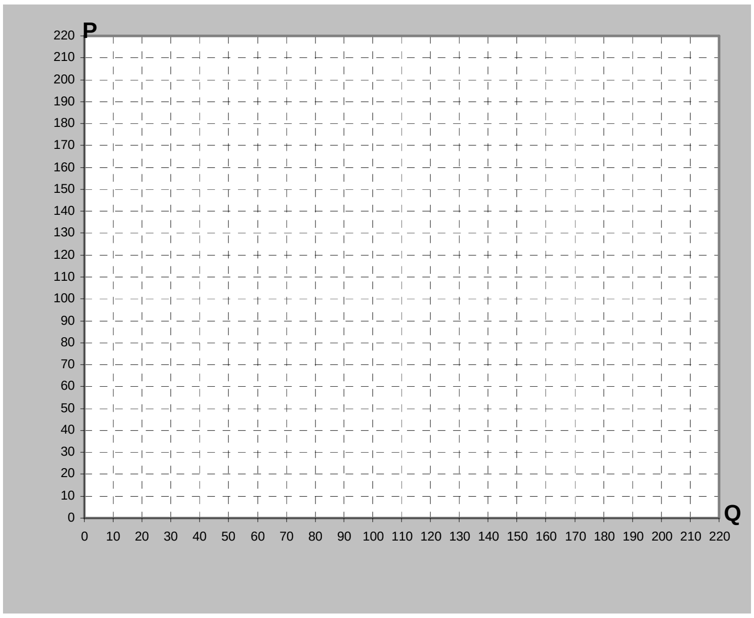
(a) Solve the pair of simultaneous equations *D* and *S* in order to obtain the equilibrium price and quantity, *P** and *Q**.

$P^* =$ ____________ $Q^* =$ ____________

(b) Invert the two equations, so that they show *P* in terms of *Q*. Make sure that your inverted equations are each the equation of a straight line.

D: S:

(c) In the figure below, plot the two lines represented by the equations obtained in (b). Label the lines *D* and *S*, and label the market equilibrium.



(d) Use the graph to work out total consumer surplus (CS) and total producer surplus (PS), and label these two areas on the graph.

$CS =$ ________ $PS =$ ________

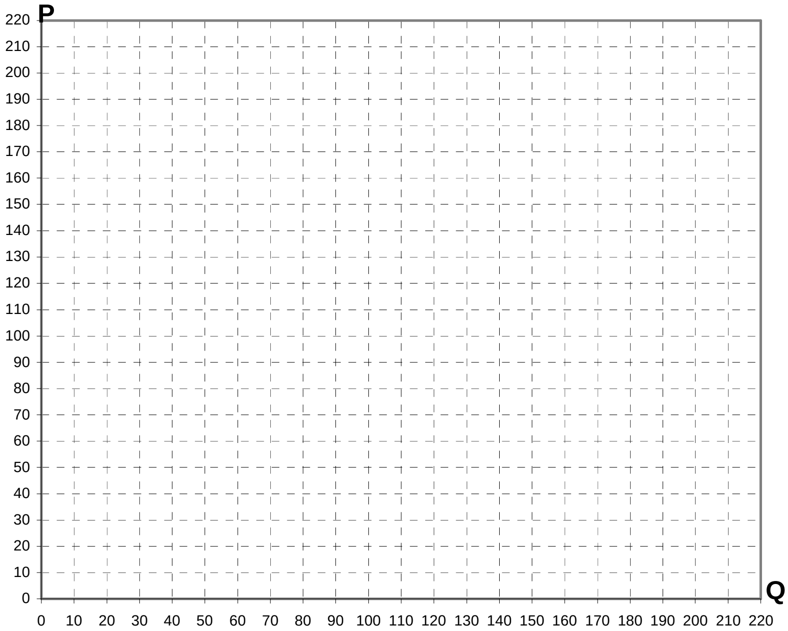
Task Two (Taxation)

Consider the Demand and Supply equations analysed in Learning Activity 3.3 above.

D: $Q = 100 - \frac{1}{2}P$ S: $Q = -100 + 2P$

Suppose the government imposes a fixed tax of £5 per unit.

(a) Calculate the effect on the equilibrium price and quantity. Demonstrate the effect in the graph below.



(b) Use the graph to calculate the amount of government revenue that results from the imposition of the tax.

Task Three (Equilibria)

Consider the following demand and supply equations:

$$D:\ P = 60 - Q + 0.6M$$

$$S:\ P = 30 + 0.5Q + 0.3C$$

Where *P* is price, *Q* is quantity, *M* is consumers' income, and *C* is labour costs.

Consider the following four combinations of *M* and *C*.

Combination	*M*	*C*	*P*	*Q*
A	100	100		
B	200	100		
C	100	200		
D	200	200		

(a) Calculate the equilibrium price and quantity for each, and insert these numbers in the final two columns of the table.

(b) Draw a demand and supply diagram showing the four equilibria.

Economic Applications - Supply and Demand: Equilibria, Consumer and Producer Surplus, Revenue and Taxation - ANSWERS

ACTIVITY ONE

Task One

(a) P*=80, Q*=60

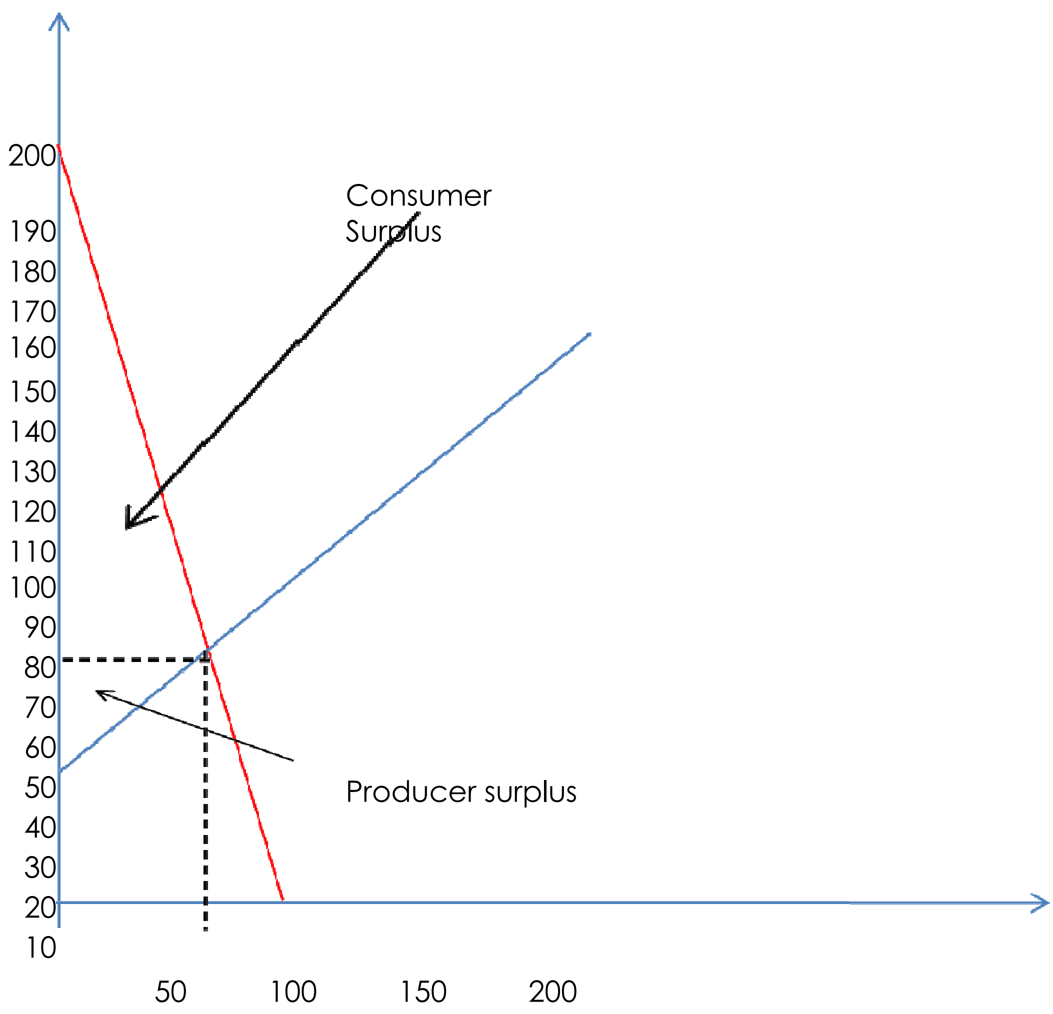
(b) Demand: P=200-2Q

Supply: P=50 +½Q

(c) see below

(d) Consumer surplus = 0.5 x 30 x 120 = 1800

Producer surplus = 0.5 x 30 x 60 = 900



Task Two

(a) Supply curve shifts to the left

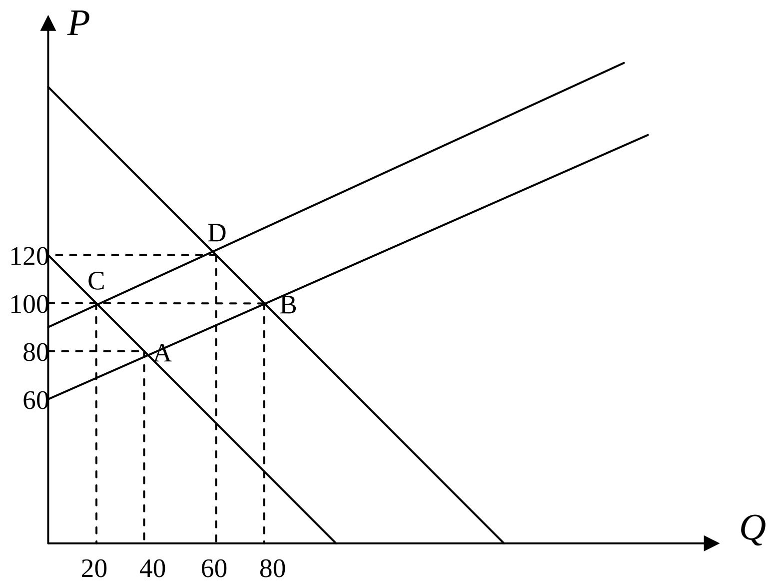
(b) Tax revenue is shown on graph.

Task Three

(a)

Combination	*M*	*C*	***P***	***Q***
A	100	100	**80**	**40**
B	200	100	**100**	**80**
C	100	200	**100**	**20**
D	200	200	**120**	**60**

(b)



Economic Applications - National Income Determination - ACTIVITIES

Learning Objectives

LO1: Students understand the meaning and significance of the marginal propensity to consumer and the multiplier

LO2: Students are able to independently calculate the MPC and the multiplier

LO3: Students are able to apply their knowledge of simultaneous equations to issues concerning the determination of national income.

TASK ONE

Each student should select one of the following values of marginal propensity to consume (*mpc*):

0.5, 0.55, 0.6, 0.66, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95

They should then be asked to carry out the following tasks, assuming their selected *mpc* value.

(a) Imagine that aggregate demand is boosted by £1million. Find the resulting first-round increase in income, second round increase, third round increase, and so on. When the increases become negligible, add them all together in order to obtain the total increase in income.

(b) Work out the multiplier directly from the *mpc*, and use this to find the income increase that results from the £1million boost to aggregate demand. Check that your answer is the same as in (a). If you have a different answer to (a), why is this?

(c) Use graph paper to confirm the answers you have obtained in (a) and (b). Assume that the aggregate demand equation is:

$$AD = 2 + cY$$

where *c* is your *mpc*. Draw this AD curve on a graph and find the equilibrium income level by locating where it crosses the 45^0 line. Now draw a new AD curve which is the original AD curve shifted upwards by one unit. Find the new equilibrium. How much has income increased? Your answer should be the same as in (a) and (b).

Economic Applications - National Income Determination - ANSWERS

(a) and (b)

The multiplier is the accurate calculation because it calculates the total change in income by taking account of all successive stages.

MPC	**Step 1**	**Step 2**	**Step 3**	**Step 4**	**Step 5**	**Step 6**	**Step 7**	**Step 8**	**Step 9**	**Step 10**	**TOTAL (m)**
0.5	£1.00	£0.50	£0.25	£0.13	£0.06	£0.03	£0.02	£0.01	£0.00	£0.00	£2.00
0.55	£1.00	£0.55	£0.30	£0.17	£0.09	£0.05	£0.03	£0.02	£0.01	£0.00	£2.22
0.6	£1.00	£0.60	£0.36	£0.22	£0.13	£0.08	£0.05	£0.03	£0.02	£0.01	£2.48
0.66	£1.00	£0.66	£0.44	£0.29	£0.19	£0.13	£0.08	£0.05	£0.04	£0.02	£2.90
0.7	£1.00	£0.70	£0.49	£0.34	£0.24	£0.17	£0.12	£0.08	£0.06	£0.04	£3.24
0.75	£1.00	£0.75	£0.56	£0.42	£0.32	£0.24	£0.18	£0.13	£0.10	£0.08	£3.77
0.8	£1.00	£0.80	£0.64	£0.51	£0.41	£0.33	£0.26	£0.21	£0.17	£0.13	£4.46
0.85	£1.00	£0.85	£0.72	£0.61	£0.52	£0.44	£0.38	£0.32	£0.27	£0.23	£5.35
0.9	£1.00	£0.90	£0.81	£0.73	£0.66	£0.59	£0.53	£0.48	£0.43	£0.39	£6.51
0.95	£1.00	£0.95	£0.90	£0.86	£0.81	£0.77	£0.74	£0.70	£0.66	£0.63	£8.03

Using multiplier

MPC	Multiplier	Change in Income £m
0.5	2	£2.00
0.55	2.2222	£2.22
0.6	2.5	£2.50
0.66	2.9412	£2.94
0.7	3.3333	£3.33
0.75	4	£4.00
0.8	5	£5.00
0.85	6.6667	£6.67
0.9	10	£10.00
0.95	20	£20.00