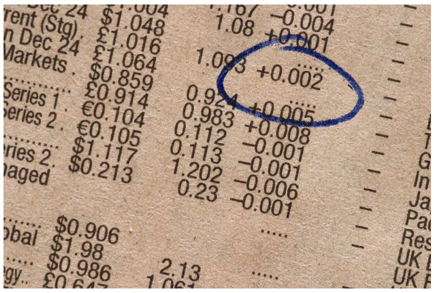
  

Teaching and Learning

Guide 3:

Linear Equations – Further Topics

Table of Contents

Section 1: Introduction to the guide

Guide 2 was concerned mainly with the equation of a straight line, and various economic settings in which such an equation is relevant. This Guide extends the analysis to settings in which there is more than one equation, and a solution to the system of equations is required.

In Sections 2 and 3 respectively, the problem of solving the system of equations graphically and algebraically is approached. This includes a discussion of “problem” cases in which there is no solution or an infinite number of solutions to a system.

In Section 4, a system of equations is solved to find the equilibrium in a market or an economy. In this Guide, examples are taken from both micro and macroeconomics. The microeconomic example is the standard demand and supply model, in which demand and supply equations are provided and the market clearing equilibrium is found. Apart from solving a system of equations, many other learning outcomes follow from this analysis. For example, the ability to invert equations is important, in order, for example, to move between demand functions and “inverse” demand functions. The computation of measures such as total revenue, consumer surplus and producer surplus involve finding the area of a region of a graph, and these skills are rehearsed thoroughly in this context. The procedure for measuring the impact of a tax on the market equilibrium is also discussed, an important issue here being that students often find it hard to understand why a tax amounts to a vertical shift in the supply curve.

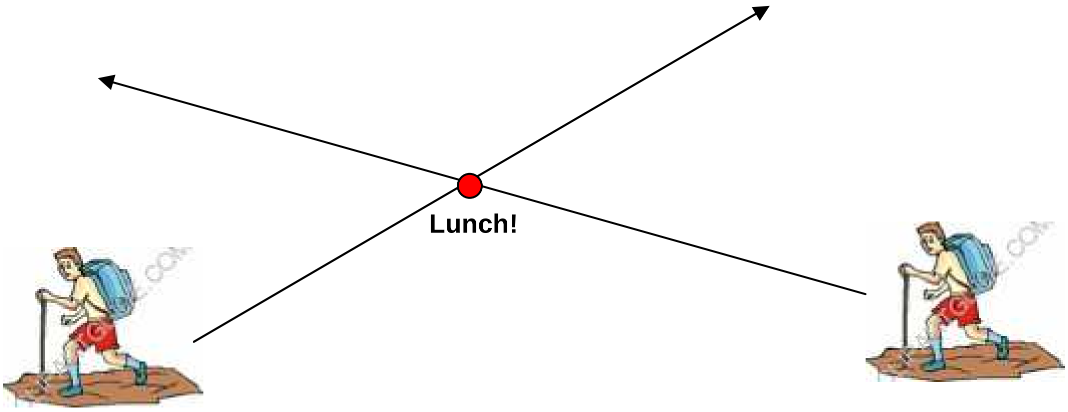
The macroeconomic example is the well-known “Keynesian cross”. This is the diagram that presents a simple but useful, account of the functioning of an economy. Algebraic solution of this problem reveals the well-known relationship between the multiplier and the marginal propensity to consume. A feature of this Guide is that the various economic concepts being covered are integral to the mathematical analysis that forms the theme. This is useful in the sense that students derive external benefits in terms of economic knowledge while practising the mathematical techniques in the ways suggested.

Section 2: Solving simultaneous equations using graphs

1. The concept of solving simultaneous equations using graphs

Students will probably have an understanding of the term, “equation” and they should have a solid grasp of graphing functions. Few will have any substantive understanding of what is meant by ‘simultaneous equations’ and it might be useful for this concept to be explained at the outset.

A straightforward to present this concept is to offer a simple example. For example, by plotting two hikers who start from different points and whose path can be illustrated by a straight line (i.e. an equation). If they want to meet for lunch they will need to see at what point and at which time their paths will cross. A simple diagram might help to clarify this problem:



This can be extended to an economic example such as trying to work out when a fixed interest rate mortgage would need to be moved to a variable one to reflect when the Bank of England lowered interest rates to a certain level.

2. Presenting the concept of solving simultaneous equations using graphs

Students will need some formal material to cover the essential concepts as well as lots of chances to apply their knowledge and understanding. An introduction to the solution of simultaneous equations could proceed along the following lines.

In Guide 1, we saw examples in which we needed to solve for an unknown, *x*. For example:

$$2x+3=9$$

Now, consider the problem of solving for two unknowns, *x* and *y*. Suppose we have the equation:

$$y=-2+2x$$

As we now know from Guide 2, this is the equation of a straight line with intercept -2 and slope 2. Suppose that someone asks us to solve for the two unknowns *x* and *y* in this equation. We would have to say that the solution is indeterminate. Why? Because any point on the straight line implied by the equation represents a solution to the equation.

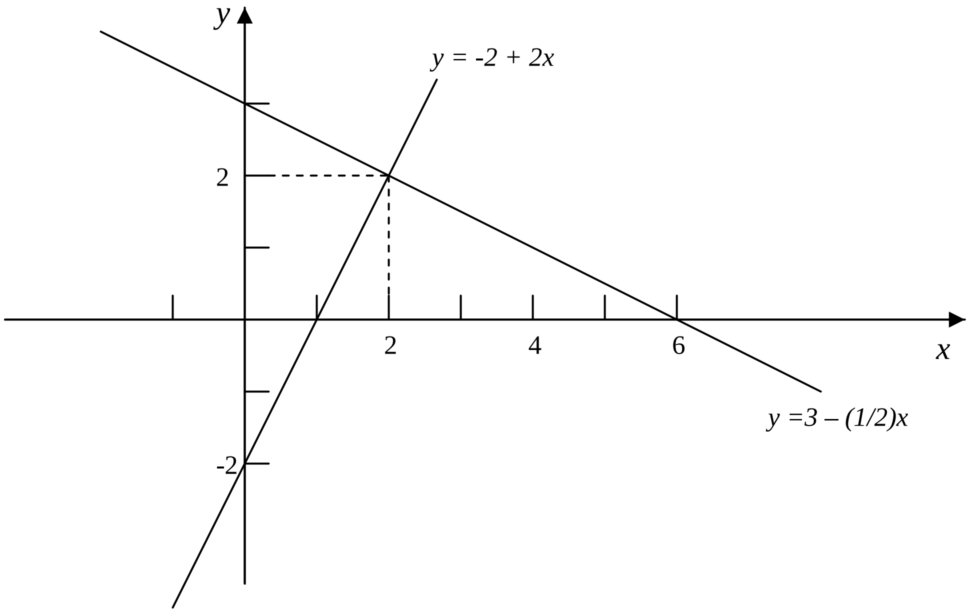
That is why, whenever we try to solve for two unknowns, we require two equations. So, suppose we have:

$$y=-2+2x \qquad (1)$$

$$y=3-\frac{1}{2}x \qquad (2)$$

These are known as a pair of simultaneous equations in the unknowns *x* and *y*. Both equations represent straight lines on a graph.

Figure 3.1: Using a graph to solve a pair of simultaneous equations



We see that the intersection of the two straight lines is at the point (2, 2), and this is the solution of the simultaneous equations: $x = 2$; $y = 2$.

3. Delivering the concept of solving simultaneous equations using graphs to small or larger groups

Students in a large setting could follow a simple set of Powerpoint slides but then have opportunities to work through problem sets, perhaps in a tutorial setting. These problem sets could also include tasks which require students to research real world examples of simultaneous equations and to report these back to the rest of the group either as a presentation or as a word processed handout.

Some examples could include:

(a) working out equilibrium prices in markets (and this could lead into a discussion of how market prices change through the dynamic interaction of supply and demand)

(b) identifying optimal points e.g. the point of tangency between an indifference curve and a budget line

(c) graphing the relationship between interest rates and present values and their intersection (See Guide 5)

(d) how cost benefit analysis seeks to help policymakers equate the economic ‘pros and cons’ of a development and, implicitly, tries to ascertain the point where costs and benefits are equal.

Links with the online question bank

The online question bank for this material can be found at http://www.metalproject.co.uk/METAL/Resources/Question_bank/Algebra/index.html. Students might find it useful to review the questions on linear functions at http://www.metalproject.co.uk/METAL/Resources/Question_bank/Algebra/index.html and then progress to those focused specifically on simultaneous equations at http://www.metalproject.co.uk/METAL/Resources/Question_bank/Algebra/index.html.

Video clips

The video clips for this section can be located at http://www.metalproject.co.uk/Resources/Films/Linear_equations/index.html. Video clip 2.06 offers an interesting application of simultaneous equations to the French wine industry. This clip can be complemented by the higher level material offered in clips 2.07 and 2.08.

4. Discussion Questions

Students could reflect on some of the practical problems that sometimes occur when economists try to specify an exact relationship between two variables. Can a ‘perfect mathematical relationship’ ever exist? What problems might this create for policymakers and businesspeople that have to make decisions. This could flag up broader issues surrounding imperfect information and information asymmetry.

For example, the Government conducted a cost benefit analysis of the Eurotunnel. We can assume that it was evaluated that the benefits of the scheme would outweigh the costs (and simultaneous equations can be introduced here) but it later transpired that the construction costs were massively underestimated. The video clip 3.08 at http://www.metalproject.co.uk/Resources/Films/Mathematics_of_finance/index.html gives an overview of the Eurotunnel investment. A good article on the problem of forecasting large public projects can be found at http://www.publicpurpose.com/pp-infra.htm

5. Activities

Learning Outcomes

LO1: Students learn how to solve simultaneous equations

LO2: Students learn how simultaneous equations can help to solve economic problems

TASK ONE

This is a learning activity that could be run in a small group. The group could be divided into three subgroups. Each subgroup would be assigned one of the following three systems of equations in *x* and *y*, and would be asked to present to the whole group their answer to the questions that follow.

System A

$$y = 1 + x \qquad (1)$$
$$y = 3 + x \qquad (2)$$

System B

$$y = 1 + x \qquad (1)$$
$$y = 3 - x \qquad (2)$$

System C

$$y = 1 + x \qquad (1)$$
$$y = 1 + x \qquad (2)$$

For the system of equations that you have been allocated, sketch a graph showing the lines represented by the two equations. In each case, use the graph to determine whether the system has:

(i) A unique solution

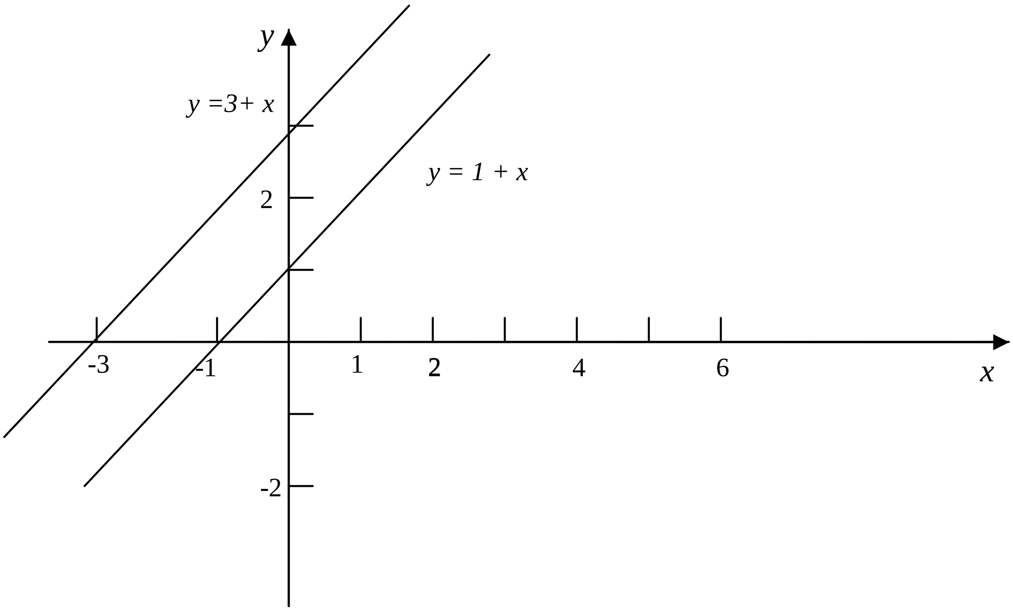
(ii) No solution

(iii) An infinite number of solutions

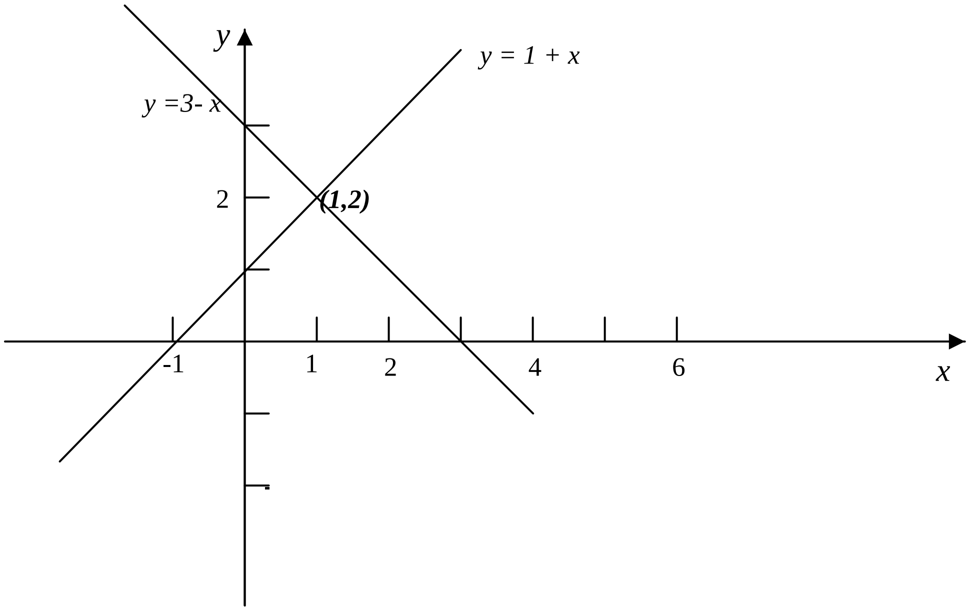
For the system which has a unique solution, solve for *x* and *y*.

ANSWERS

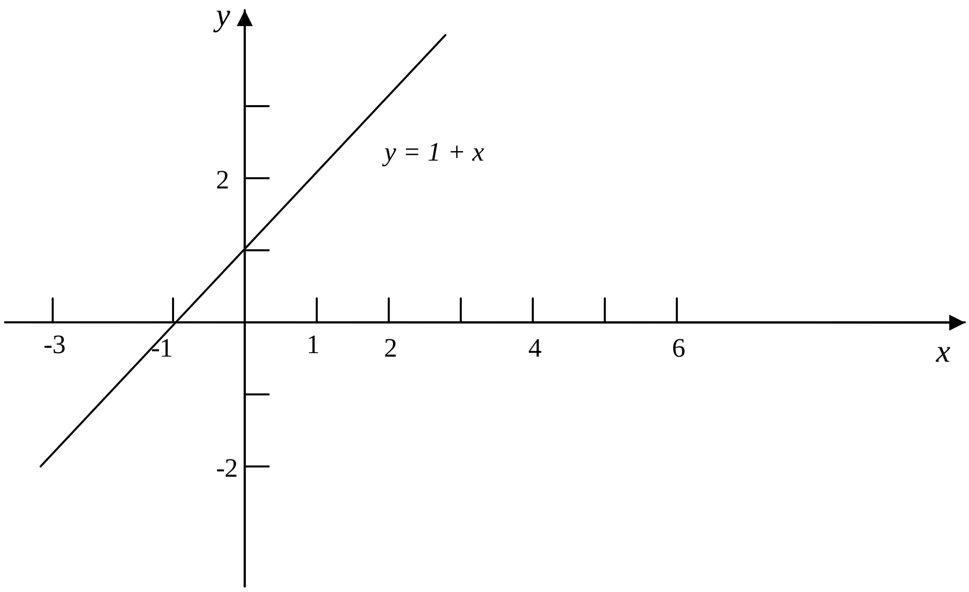
System A



System A has no solutions



System B has a solution at (1,2)



System C has an infinite number of solutions: in effect every point on the line offers a solution because the two equations are identical

6. Top Tips

Creating opportunities for students to work in small teams or pairs is a good way for them to demonstrate their understanding to others – and so build their mathematical self-confidence and self-esteem – as well as providing scope for students to help each other. This could be particularly useful in a 'hard topic' such as simultaneous equations.

7. Conclusion

Lecturers will probably want to start with a theoretical overview together with worked examples and give students lots of occasions to practice this, both supervised and as part of private study.

Section 3: Solving simultaneous equations using algebra

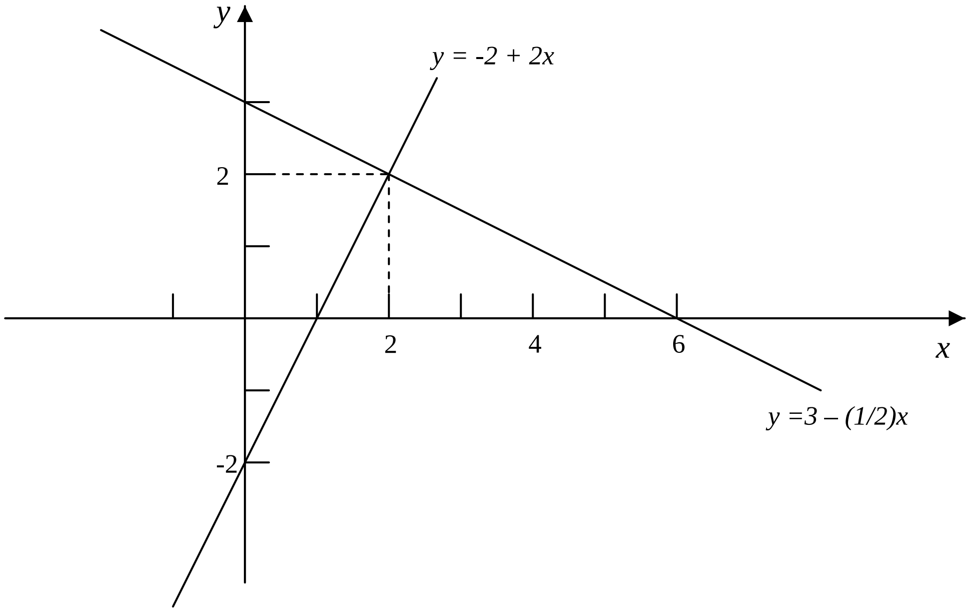
1. The concept of solving simultaneous equations using algebra

At this stage it would probably be a good idea to remind students that a graphical solution is simply a mathematical picture. Students will have graphed functions before and the next step is to ‘reverse engineer’ and move from the graphical back to the algebraic. Lecturers might find it beneficial to students some linear graphing tasks to complete as preparatory work.

2. Presenting the concept of solving simultaneous equations using algebra

Lecturers could find it useful to return to the earlier example (see Section 2 above) and solve the same problem using algebra. This approach could help to bolster the confidence of weaker students who can see for themselves that the (harder) algebraic solution is simply the flipside of the (easier) graphical solution they had already encountered. Students often need this reassurance: they can often feel comfortable with descriptive and graphical approaches but then believe the algebra is beyond them.

So, using this approach we can obtain the solution of the original question using algebra. We had the following diagram:



Both equations show expressions for *y*, so we can just set the two right-hand sides equal:

$$-2+2x=3-\frac{1}{2}x$$

We then solve for *x* in the usual way:

$$2x+\frac{1}{2}x=3+2$$
$$\therefore \frac{5}{2}x=5$$
$$\therefore x=\frac{2}{5}\times 5=\underline{\underline{2}}$$

Having solved for *x*, we then solve for *y* by putting our solution for *x* back into equation (1):

$$y=-2+2(2)=\underline{\underline{2}}$$

Having solved for both *x* and *y*, we can check the solution by putting both back into equation (2):

$$2=3-\frac{1}{2}(2)$$

which is true, confirming that the solution *x* = 2, *y* = 2 is correct.

Note that this solution that we have obtained using algebra is the same as the one obtained using the graph.

3. Delivering the concept of solving simultaneous equations using algebra to small or larger groups

Students will want to practise working through problem sets where they are solving simultaneous equations. Students could work together creating their own worksheets with questions and answers which are then circulated within the group. In this way, a large body of student led work can be produced which underlines students knowledge and understanding and also gives them a way to help test the understanding of their peers. This also creates obvious opportunities for peer assessment. Lecturers will want to verify that questions and answers are correctly before they are circulated to others in the group.

Links with the online question bank

As noted above, the online question bank for this material can be found at http://www.metalproject.co.uk/METAL/Resources/Question_bank/Algebra.index.html.
Students might find it useful to review the questions on linear functions at
http://www.metalproject.co.uk/METAL/Resources/Question_bank/Algebra/index.html
and then progress to those focused specifically on simultaneous equations at
http://www.metalproject.co.uk/METAL/Resources/Question_bank/Algebra/index.html.

Video clips

As noted above, the video clips for this section can be located at http://www.metalproject.co.uk/Resources/Films/Linear_equations/index.html. Video clip 2.06 offers an interesting application of simultaneous equations to the French wine industry. This clip can be complemented by the higher level material offered in clips 2.07 and 2.08.

4. Discussion Questions

Students could consider and share examples of 'personal simultaneous equations'. For example, the simple trade-off between working longer hours in a part-time job against the marks they receive for dissertations and their degree class.

5. Activities

ACTIVITY ONE

Learning Objectives

LO1: Students learn how to recognise and specify simultaneous equations

LO2: Students learn how to independently solve simultaneous equations

LO3: Students learn the special case where there are no solutions or an infinite number of solutions

Task One

Solve:

$$3x + 2y = 5 \quad (1)$$
$$x - 3y = 9 \quad (2)$$

Task Two

This Activity can be run in a small group, by dividing students into three subgroups with each subgroups being assigned to one of three systems, which they are then asked to solve using algebra, and also to present their solution.

(A)
$$4x + 2y = 2 \quad (1)$$
$$-5x - y = 2 \quad (2)$$

(B)
$$2x - 5y = 20 \quad (1)$$
$$x + 4y = -3 \quad (2)$$

(C)
$$3x - 2y = -1 \quad (1)$$
$$x + y = 8 \quad (2)$$

Use algebra to solve the system of equations that you have been allocated.
Verify that your solution is correct by substituting into the equation that you have not used.

Task Three

As an economist you find two economic relationships between income (y) and consumption (x) as follows:

Observation 1: y=- -2 + 2x

Observation 2: y= 2x

(a) Graph these two equations and solve. What can you conclude about the type of function relationship and the problem you had trying to solve?

You find two more observations:

Observation 3: y= -2 + 2x

Observation 4: y -4 + 4x

(b) Graph these two functions and try to solve. What do you notice?

ANSWERS

ACTIVITY ONE

Task One

We first need to multiply every term in equation (2) by 3:

$$3x - 9y = 27 \qquad (3)$$

We are then able to eliminate *x* by subtracting equation (3) from equation (1):

(1)-(3) $\qquad 11y = -22$

$$\therefore \underline{\underline{y = -2}}$$

Then substitute the solution for *y* into (1):

$$3x + 2(-2) = 5$$
$$\therefore 3x - 4 = 5$$
$$\therefore 3x = 9$$
$$\therefore \underline{\underline{x = 3}}$$

Finally, we may check the solution *x* = 3, *y* = -2, using (2).

Task Two

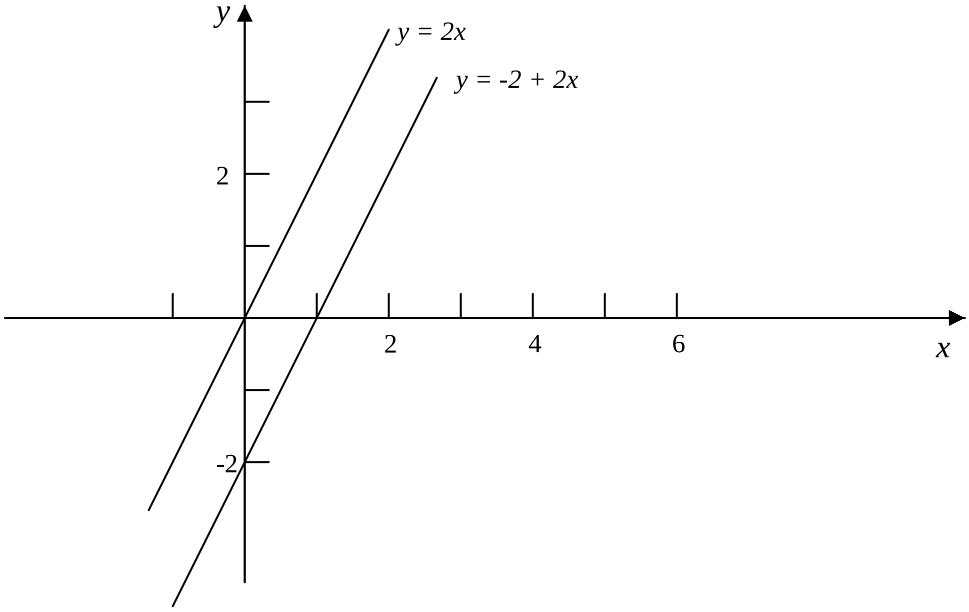
(A) x=-1, y=+3

(B) x=+5, y=-2

(C) x=+3, y=+5

Task Three

(a)



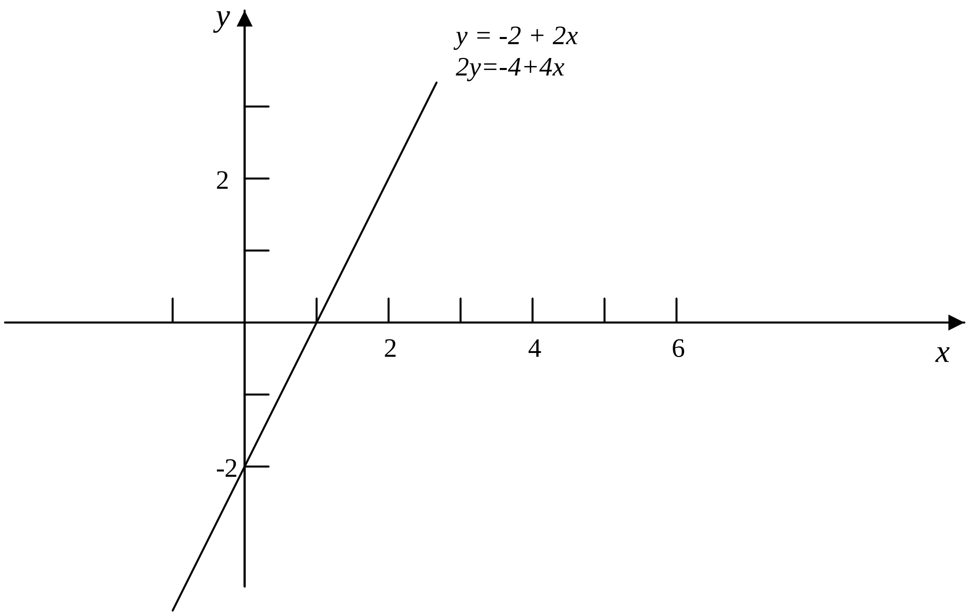
We have a situation in which the two lines run parallel to each other, that is, the two equations have the same slope but different intercepts. It is well known that “parallel lines never meet”, so such a pair of equations has <u>no solution</u>.

This problem case is illustrated in Figure 3.2, in which the two equations are:

$$y = -2 + 2x$$
$$y = 2x$$

Because these two lines have the same slope (2), but different intercepts, they are parallel, implying that there is no solution to the pair of equations represented by the two lines.

(b)



Here the two lines coincide. This would happen if the two equations had the same slope and the same intercept, i.e. they are the same equation. In this situation, every point on the line is an intersection of the two lines, and therefore every point on the line represents a solution to the pair of equations. In this situation, we say that the pair of equations has an infinite number of solutions.

6. Top Tips

Ask students to explain verbally why the ‘problem cases’ of parallel functions or identical functions exist. Often students can reveal a lack of depth of understanding when they cannot articulate answers clearly and concisely. Equally, students who can give cogent and confident answers will be able to show they have understood the practical implications of these two special cases.

7. Conclusion

This can be a ‘hard topic’ and lecturers will want to ensure that all students are able to make the intellectual jump from graphical to algebraic solutions. Time might need to bet set aside within a scheme of work for lower ability students to be able to receive more one-to-one support. It might be possible and desirable for higher ability students to work with lower ability students in other groups to help support them whilst at the same time consolidating the understanding of the higher ability individuals: often students who are supporting others can themselves be beneficiaries.

Section 4: Economic Applications - (A) Supply and Demand: Equilibria, Consumer and Producer Surplus, Revenue and Taxation

1. The concept of supply and demand

This is a good example to use both to consolidate students’ understanding of supply and demand but also to strengthen their mathematical understanding and capacity to independently apply a technique to an economic problem.

2. Presenting the concept of supply and demand

This guidance is intended to provide a logical and structured sequence of material which covers supply and demand and the associated topics of equilbria, consumer and producer surplus, revenue and taxation. Lecturers will probably find it useful to follow the outline below.

(a) Supply and demand

A mathematical treatment of demand and supply is best motivated using a straightforward example such as the one that follows.

One of your activities is handcrafting “silver” rings. Sometimes, you set up a stall and attempt to sell them. You know from experience that:

> if you set a price of £1 per ring, you sell eight rings;
> if you set a price of £2, you sell seven;
> if you set a price of £3, you sell six;

if you set a price of £4, you sell five;

:

:

if you set a price of £9, you do not sell any!

Let *P* be the price that you set, and let *Q* be the number that you sell (*Q* standing for Quantity). The information presented above is represented by the following equation:

D: $Q = 9 - P$

This equation is known as the demand equation for your silver rings. It shows how many will be sold (*Q*) when the price is *P*.

What price will you choose to set? Well, actually, you don't choose the price; you let the market choose it for you. In order to find what price will emerge, we need to consider the supply equation as well.

If you are offered only £1 per ring, you are not willing to spend time producing them, so you do not produce any. If you are offered £2, you are willing to produce only one. If you are offered £3, you are willing to produce two; and so on. If you are offered £10, you produce nine.

So, if we again let *P* be the price, but this time *Q* is the number you produce, the equation implied by the information just given is:

S: $Q = -1 + P$

This equation is known as the supply equation for your silver rings. It shows how many you are willing to produce (*Q*) when the price is *P*.

So, how many are sold, and at what price? We need to find the price at which the number demanded equals the number supplied.

$$9 - P = -1 + P$$
$$\therefore 10 = 2P$$
$$\therefore \underline{\underline{P^* = 5}}$$

Notice this is the only price at which the number of rings that customers wish to purchase is equal to the number of rings that you are willing to produce. For this reason, it is called the equilibrium price. It is the price which will prevail if the market is allowed to function freely. We denote the equilibrium price as *P**.

How many rings are traded in equilibrium? We can answer this by substituting the equilibrium price *P** = 5 back into the demand equation (or the supply equation):

$$Q^* = 9 - 5 = \underline{\underline{4}}$$

So, in equilibrium, 4 rings are traded, at a price of £5 each.

(b) Linking to Supply and Demand Curves, Surplus and Revenue

The preceding material can easily be extended to cover these topics.

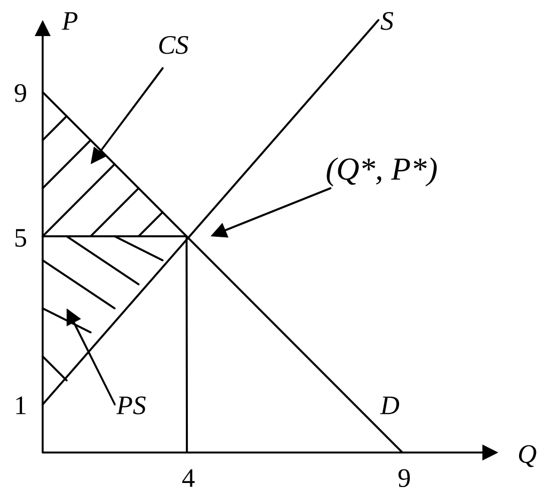
Students will need to be reminded of graphing conventions such that we show quantity (Q) on the horizontal axis, and price (P) on the vertical but that in order to do this, we first need to invert the two equations specified above.

That is:

Demand:
$$Q = 9 - P$$
$$\therefore \underline{P = 9 - Q}$$

Supply:
$$Q = -1 + P$$
$$\therefore \underline{P = 1 + Q}$$

And then to create the traditional supply and demand diagram:



Students could be reminded that:

(c) Linking to total revenue

Having computed the equilibrium price and quantity, it is a simple matter to deduce the revenue earned by the producer or producers. This is simply the product of price and quantity. So, in the example above, total revenue is:

$$Total\ Revenue = P^* \times Q^* = 5 \times 4 = \underline{\underline{20}}$$

with total revenue represented by an area on the graph by the area of the rectangle with two sides the axes and the other two sides drawn parallel to the axes and passing through the equilibrium point.

(d) Linking to consumer surplus and producer surplus

(i) Consumer Surplus

The above figure can again be reproduced to remind students of the original example where there was one customer who was prepared to pay £8 for a ring and that this customer only pays the equilibrium price of £5 and so therefore we say that this customer extracts consumer surplus of £3. Likewise, there was another customer who was prepared to pay £7, but only pays £5, thus extracting consumer surplus of £2. The customer who was prepared to pay £6 extracts consumer surplus of £1. The customer who was prepared to pay £5, pays

exactly this price, and extracts no consumer surplus. The remaining customers are not prepared to pay the equilibrium price, and therefore go without the product.
This can be summarised as: consumer surplus being conventionally computed as the area of the triangle enclosed by the demand curve, the vertical axis, and the horizontal line at *P**.

To find this area, students can use ideas introduced in Guide 2 where the triangle area is half of a square with sides 4. Its area is therefore:

$$\frac{1}{2}\times 4\times 4 = \underline{\underline{8}}$$

Our measure of consumer surplus is therefore 8.

(ii) Producer Surplus

Next, students could focus on the supply curve by again reconsidering that the first ring produced was prepared to sell for £2, but the supplier actually received £5 and that therefore producer surplus of £3 was extracted from the sale of this first ring. The second ring was prepared to sell for £3, with £2 of producer surplus extracted, and so on.

Students will need to know that the conventional measure of total producer surplus is the area of the triangle enclosed by the supply curve, the vertical axis, and a horizontal line at *P**. This area is:

$$\frac{1}{2}\times 4\times 4 = \underline{\underline{8}}$$

and the measure of producer surplus is 8.

(e) Linking to taxation

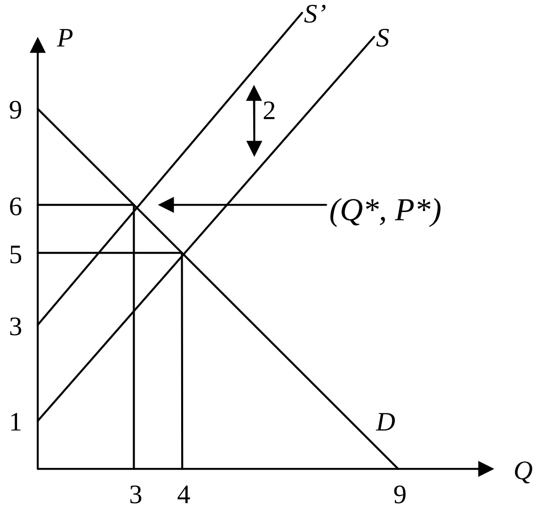
At the end of an introductory microeconomics module, many students know that the imposition of a tax on a good is represented by a parallel upward shift in the supply curve by the amount of the tax. However, there is no doubt that students find it hard to understand why a tax causes such a shift.

An explanation is provided here.

Look at the supply curve used previously (in the rings example). Imagine again that you are the producer. If you only produce one ring, the minimum price you are prepared to accept for it is £2. Now imagine that the Government imposes a tax of £2 per ring that you, the supplier, are required to pay to the taxman when the purchase is complete. Then, the minimum price you would be prepared to accept for the single ring is £4. Therefore the point on the Supply curve corresponding to Q=1 has shifted upwards from P=2 to P=4.

If you produce two rings, the minimum price you are prepared to accept per ring is £3. With the £2 tax imposed, the minimum you are prepared to accept becomes £5. Hence the point on the supply curve corresponding to Q=2 also shifts upwards by £2.

In fact, the effect of the £2 tax per unit is an upward shift of the entire supply curve by £2. Generally, the effect of a tax is an upward shift in the supply curve by the amount of the tax. The new supply curve is labelled as *S'* in the figure below. Note that the new equilibrium is shown as the point of intersection of the demand curve with the new supply curve.



In order to solve for the equilibrium using algebra, we simply note that the equation of the supply curve has changed from $P = 1 + Q$ to $P = 3 + Q$. The demand curve is still $P = 9 - Q$. We therefore set $3 + Q = 9 - Q$ which gives *Q* = 3. Substituting into the demand equation then gives $P = 9 - 3 = 6$. Hence we have established that the tax has moved the equilibrium from (4, 5) to (3, 6).

At this point, it needs to be stressed that the price increase (£1) is less than the amount of the tax (£2). This is a consequence of the demand curve being downward-sloping, and simply tells us that not all of the tax is passed on to the consumer. The steeper the demand curve, the greater the price increase resulting from the tax, and therefore the more of the tax that is passed on to the consumer.

The amount of tax raised by the government (government revenue) is obtained by simply multiplying the after-tax quantity by the amount of the tax. In the example, government revenue is $3 \times 2 = 6$.

3. Delivering the concepts to small or larger groups

This material is wholly applied and students will want to practise generating their own answers. A good way to help them consolidate their knowledge is to ask them to create a booklet which summarises:

- The key economic terms i.e. Supply, demand, producer surplus etc;
- Provides some real world examples e.g. the demand curve for coffee
- Sets out some scenarios which the student has drafted **themselves** or in given pairs together with some mathematical solutions.

The lecturer might want to allocate topics within a group so that a complete set of booklets is produced and which can be copied for each member. This creates valuable opportunities for team-working, peer assessment and independent learning. It also puts the student at the centre of the learning rather than the students being 'taught at' by the lecturer.

Links with the online question bank

Questions on supply and demand and related issues can be found at:
http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

Video clips

Video clips covering a wide range of related topics can be found at http://www.metalproject.co.uk/Resources/Films/Linear_equations/index.html. Lecturers might find the clips on equilibriums (video clip 2.05 to 2.08) the most useful when covering microeconomics.

4. Discussion Questions

Consider asking students to think about some of the following questions:

(a) Why might the competition authorities be interested in consumer surplus?

(b) Why might the Office of Fair Trading (OFT) be concerned if the market share of the top three supermarkets becomes very high? How might measurements of consumer and producer surplus be used in any investigation by the OFT?

(c) Research some products and services that the Government taxes. Why do you think the Government chose these products and services? Using a supply and demand diagram show how taxing petrol is likely to be far more effective as a tax raising strategy than blue shirts.

5. Activities

ACTIVITY ONE

Learning Objectives

LO1: Students learn how to solve simultaneous equations

LO2: Students learn how to apply their knowledge of simultaneous equations to micro- and macroeconomic problems

TASK ONE (Supply and Demand)

The Demand and Supply equations for a particular good are:

D: $Q = 100 - \frac{1}{2}P$ S: $Q = -100 + 2P$

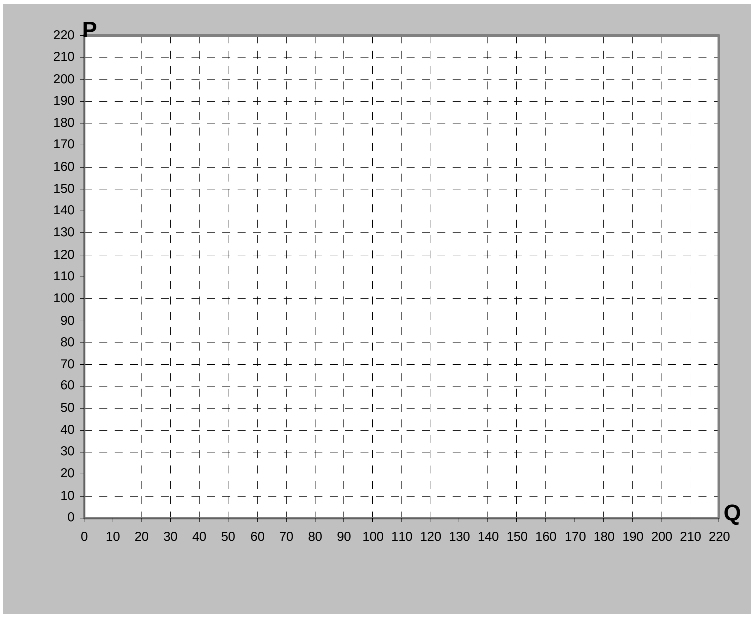
(a) Solve the pair of simultaneous equations *D* and *S* in order to obtain the equilibrium price and quantity, *P** and *Q**.

$P^* =$ ________ $Q^* =$ ________

(b) Invert the two equations, so that they show *P* in terms of *Q*. Make sure that your inverted equations are each the equation of a straight line.

D: S:

(c) In the figure below, plot the two lines represented by the equations obtained in (b). Label the lines *D* and *S*, and label the market equilibrium.



(d) Use the graph to work out total consumer surplus (CS) and total producer surplus (PS), and label these two areas on the graph.

$CS =$ ________ $PS =$ ________

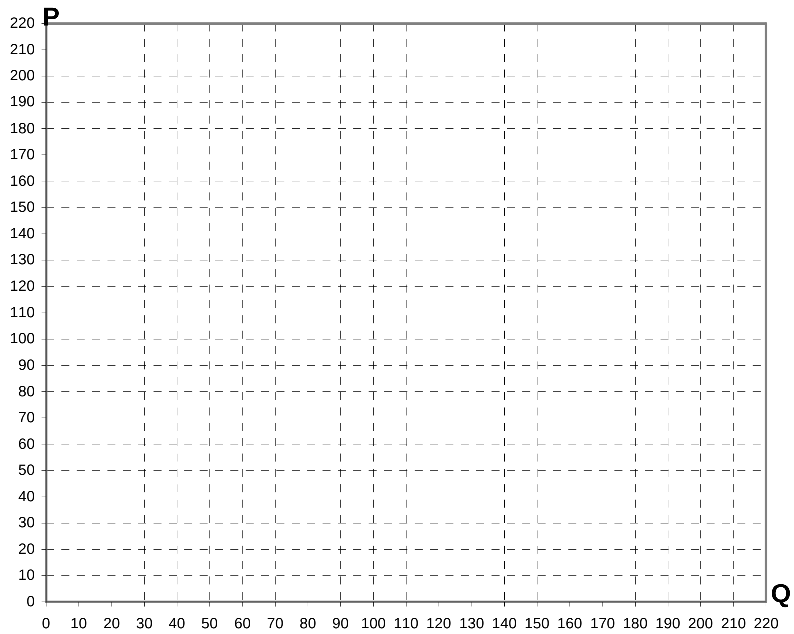
Task Two (Taxation)

Consider the Demand and Supply equations analysed in Learning Activity 3.3 above.

D: $Q = 100 - \frac{1}{2}P$ S: $Q = -100 + 2P$

Suppose the government imposes a fixed tax of £5 per unit.

(a) Calculate the effect on the equilibrium price and quantity. Demonstrate the effect in the graph below.



(b) Use the graph to calculate the amount of government revenue that results from the imposition of the tax.

Task Three (Equilibria)

Consider the following demand and supply equations:

$$D:\ P = 60 - Q + 0.6M$$

$$S:\ P = 30 + 0.5Q + 0.3C$$

Where *P* is price, *Q* is quantity, *M* is consumers' income, and *C* is labour costs.

Consider the following four combinations of *M* and *C*.

Combination	*M*	*C*	*P*	*Q*
A	100	100		
B	200	100		
C	100	200		
D	200	200		

(a) Calculate the equilibrium price and quantity for each, and insert these numbers in the final two columns of the table.

(b) Draw a demand and supply diagram showing the four equilibria.

ANSWERS

ACTIVITY ONE

Task One

(a) P*=80, Q*=60

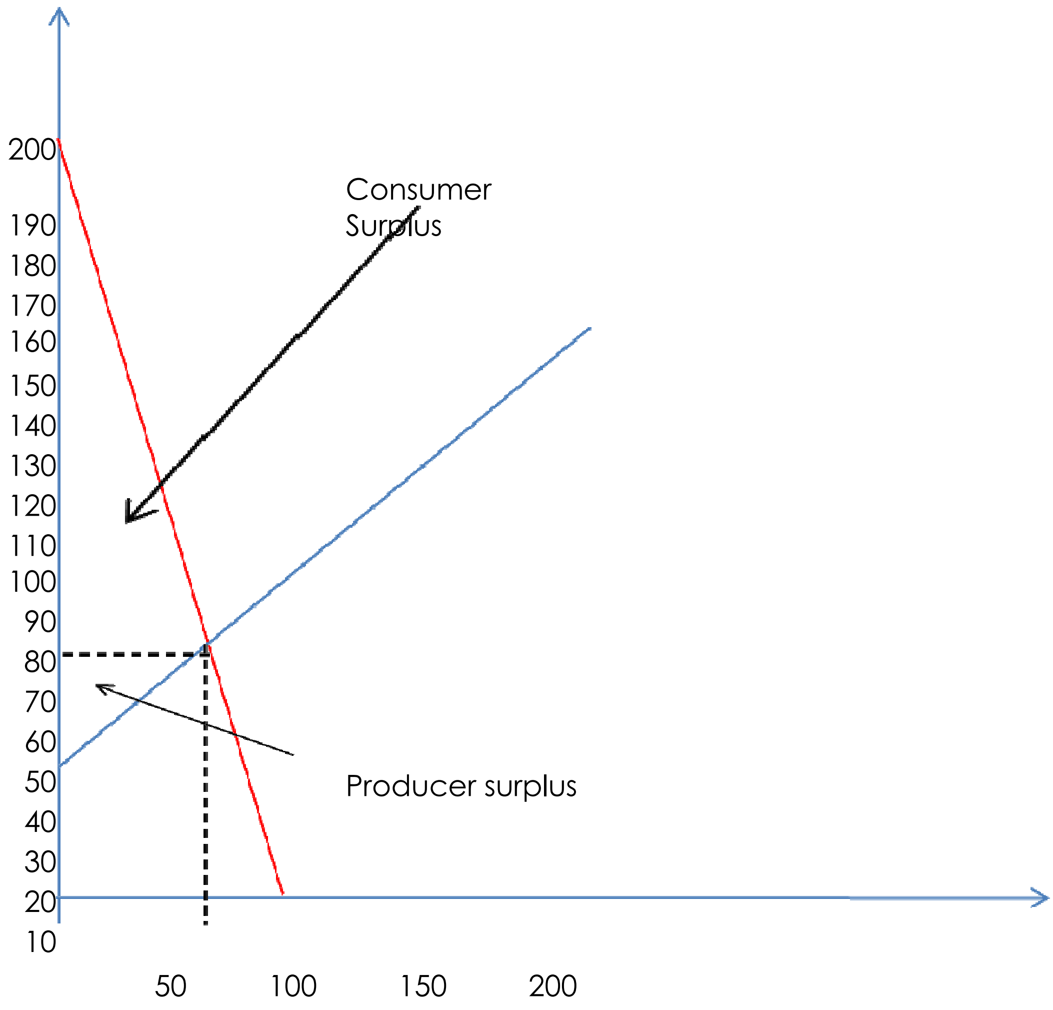
(b) Demand: P=200-2Q

Supply: P=50 +½Q

(c) see below

(d) Consumer surplus = 0.5 x 30 x 120 = 1800

Producer surplus = 0.5 x 30 x 60 = 900


200
Consumer
Surplus
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
Producer surplus
50
100
150
200

Task Two

(a) Supply curve shifts to the left

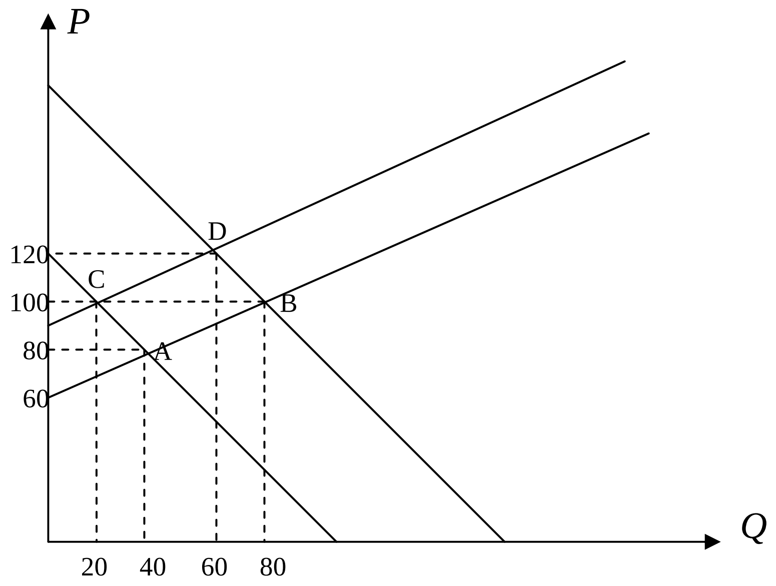
(b) Tax revenue is shown on graph.

Task Three

(a)

Combination	*M*	*C*	***P***	***Q***
A	100	100	**80**	**40**
B	200	100	**100**	**80**
C	100	200	**100**	**20**
D	200	200	**120**	**60**

(b)



6. Top Tips

Encourage students to bring in their own examples of equilibrium prices moving e.g. on-line auctions such as e-bay, or from a 'quality' economics news article, e.g. The Economist. Always try to contextualise the mathematics.

7. Conclusion

This part tries to bring together the threads of simultaneous equations to illuminate microeconomics problems. Lecturers will probably find it more effective to start with a theoretical and conceptual overview but then to supplement with plenty of practical 'hands-on' exercises in tutorials. Most students will probably find to easier to start with the microeconomic examples before moving on to the macroeconomic applications (see below).

Section 4: Economic Applications - (B) National Income Determination

1. The concept of national income determination

The principle that is central to this part of the Guide is the multiplier, which shows that a boost in aggregate demand normally results in an income increase that is greater than the initial boost. The really important point about the multiplier is that it can be worked out and analysed in three different ways: dynamic analysis; comparative static analysis; graphical analysis.

2. Presenting the concept of national income determination

The topic should start with a straightforward motivational example, along the following lines.

Step 1:

What happens if Government spending rises by £1 billion, i.e. if the Government injects £1bn into economy, by e.g. raising unemployment benefits?

A rise in Government spending generates an immediate rise in total household income of the same amount. This is obvious: if the Government gives £1bn to households, households will receive £1bn! However, this is not the end of the story.

Step 2:

The next thing is, what will the households do with the £1bn? Naturally, they will spend some of it, and save the rest. Let's assume they spend three quarters of it [in economic terms, we would say that the marginal propensity to consume (mpc) is 0.75.] So, the households spend

£0.75bn. What happens to the £0.75bn? It goes into the pockets of the employees of firms producing the goods. What do they do with the £0.75bn? They spend three quarters of it, £0.56bn, and save the rest, and so on, ad infinitum.

What is the total effect on the economy of the injection of £1bn? Answer:

$$\pounds[1+0.75+0.56+0.42+0.32+0.24+0.18+0.13+0.10+0.07+0.06+0.04+\cdots]bn$$
$$=\underline{\pounds 4bn}$$

Thus, the injection of £1bn into the economy has brought about an increase in Income of £4bn. Since the eventual increase in income is four times the initial boost, we say that the <u>multiplier</u> is 4.

Step 3:

Students can then move on to a formal economic analysis.

Total expenditure (or "aggregate demand") is:

$$AD = C + I + G \qquad (1)$$

where *C* = total spending by households ("Consumption")

I = total spending by firms (on capital equipment); also known as "Investment".

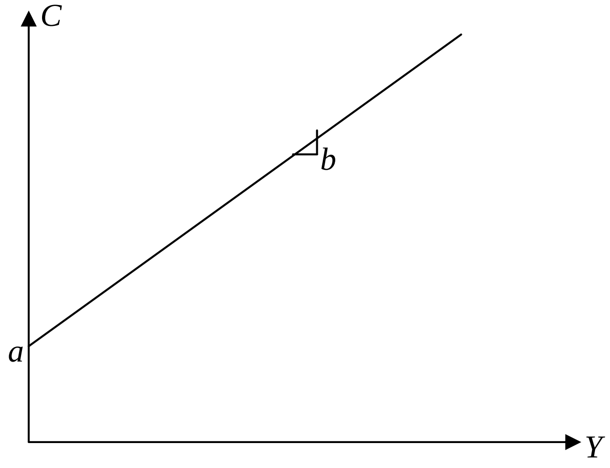
G = total spending by Government

AD = total spending by households, firms and Government; also known as "Aggregate Demand".

The first part of this, Consumption (C) depends on income (Y), according to:

$$C = a + bY \qquad (2)$$

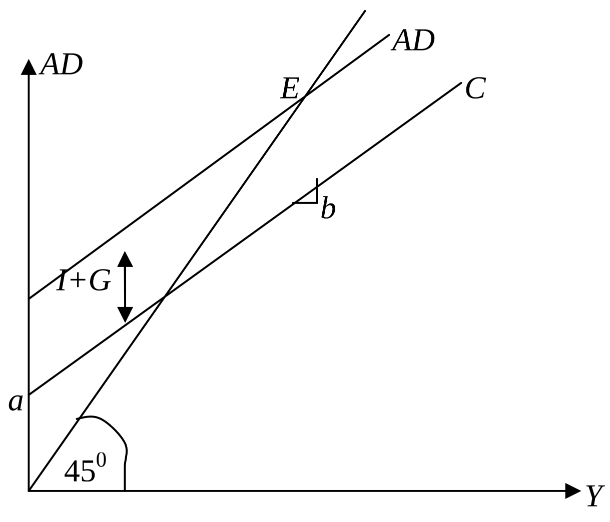
This is the <u>Consumption Function</u>. A graph of the consumption function looks like this:



a is the level of autonomous consumption, and *b* is the *marginal propensity to consume*.

Step 4:

The other components of Aggregate Demand, *I* and *G*, are assumed to be exogenous, i.e. they do not depend on income, *Y*. Hence, a graph of aggregate demand is simply the consumption function shifted upwards by *I* + *G*:



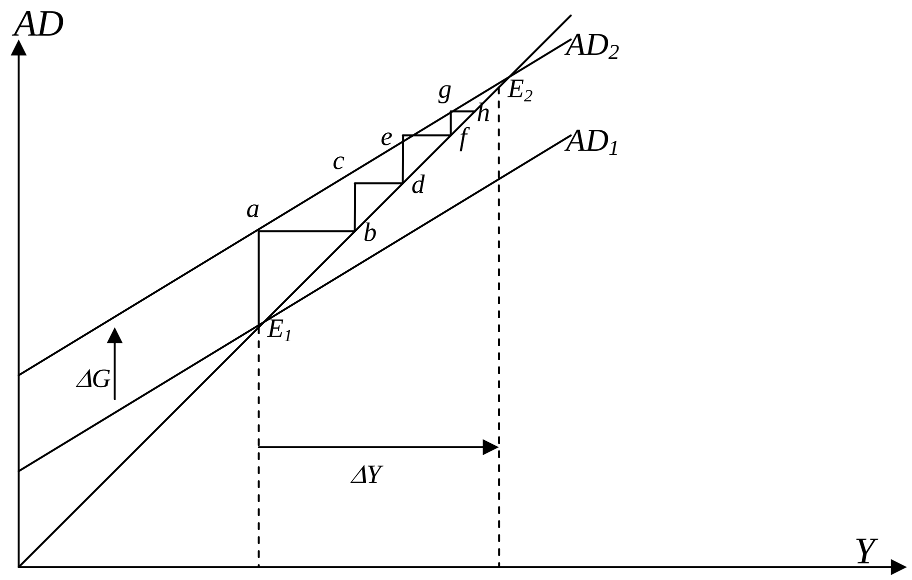
Remember the Engel Curve (Guide 2), which shows expenditure against income for a single household. The Aggregate Demand function is a bit like one big Engel curve, with an important difference: we can only be at one point on the Aggregate Demand function, because total spending in the economy must be equal to total income earned, that is:

$$AD \equiv Y, \text{ or } Y \equiv AD. \qquad (3)$$

The three lined symbol ($\equiv$) in (3) indicates that the relationship between the two quantities is an identity. This is more than just an equality: for an identity, it is always the case that the two quantities are equal.

Since *AD* and *Y* must be equal, we must be on the 45^0-line in Figure X above. Therefore the equilibrium level of income in the economy is given by the point *E*, where the aggregate demand curve crosses the 45^0-line.

Returning to the question we are asking, we need to consider what happens to the graph when *G* rises. Referring to Figure Y below, let us assume that aggregate demand is initially given by AD_1 and we are initially at equilibrium E_1. Let us next assume that *G* rises by an amount ΔG. this causes an upward shift in the Aggregate Demand curve from AD_1 to AD_2, and we might imagine that this takes us temporarily off the 45^0-line to point *a*. The resulting first-round increase in income is equal to ΔG and takes us from *a* to *b*. Consumers, as a result of now being better off by the amount ΔG, spend a certain proportion of this, taking us from *b* to *c*. The resulting second-round increase in income takes us from *c* to *d*. The process continues until we reach the point E_2, which is, of course, the point of intersection between the new aggregate demand curve (AD_2) and the 45^0-line.



Step 5:

Obtaining the multiplier using algebra

(1) tells us that: $AD = C + I + G$

While (3) tells us that: $AD = Y$

Essentially, what we have here is two simultaneous equations in the unknowns *AD* and *Y*. This is the main reason why this topic fits in with the theme of the Guide.

Combining the two equations, we obtain:

$$Y = C + I + G \qquad (4)$$

Inserting the consumption function (2) into (4):

$$Y = a + bY + I + G \qquad (5)$$

Collect terms:

$$Y - bY = a + I + G$$
$$\therefore (1-b)Y = a + I + G$$
$$\therefore Y = \frac{a}{1-b} + \frac{I}{1-b} + \frac{G}{1-b}$$

We could write this last line as:

$$Y = \frac{a}{1-b} + \frac{1}{1-b}I + \frac{1}{1-b}G \qquad (6)$$

From (6), it is clear that, if *G* rises by £1 billion, *Y* will rise as a result by:

$$£\left(\frac{1}{1-b}\right)billion \qquad (7)$$

Thus we see that the multiplier is always <u>one over one minus the marginal propensity to consume (mpc)</u>:

$$multiplier = \frac{1}{1-mpc}$$

In the numerical example, the mpc was 0.75, so the multiplier is:

$$\frac{1}{1-0.75} = \frac{1}{0.25} = \frac{1}{1/4} = \underline{\underline{4}}$$

which is in agreement with the previous analysis.

3. Delivering the concept of national income determination to small or larger groups

The above steps are intended to provide a straightforward series of lectures or episodes which link linear mathematics to contemporary economics. Students could be encouraged to prepare their own presentations which were delivered in a tutorial setting. Assessment criteria could be agreed in advance within the group and each presentation then judged by each students' peers. Assessment criteria could include: accuracy (i.e. Is the economic explanation sound?), use of resources, clarity, use of voice, interest, application to the ‘real world’ etc.

Links with the online question bank

Lecturers could relate much of this material to the applied economics questions located at http://www.metalproject.co.uk/METAL/Resources/Question_bank/Economics%20applications/index.html

Video clips

There are useful links at http://www.metalproject.co.uk/Resources/Films/Mathematical_review/index.html which include explanations of the multiplier effect (video clip 1.11) and the balanced budget multiplier (video clip 1.12)

4. Discussion Questions

Students could research macroeconomic data (e.g. back page of The Economist) and explain how changes in key variables such as consumption, investment and interest rates could influence levels of national income and employment. This could lead into broader areas such as inflation and international trade e.g. exchange rates and balance of trade and payments.

5. Activities

Learning Objectives

LO1: Students understand the meaning and significance of the marginal propensity to consumer and the multiplier

LO2: Students are able to independently calculate the MPC and the multiplier

LO3: Students are able to apply their knowledge of simultaneous equations to issues concerning the determination of national income.

TASK ONE

Each student should select one of the following values of marginal propensity to consume (*mpc*):

0.5, 0.55, 0.6, 0.66, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95

They should then be asked to carry out the following tasks, assuming their selected *mpc* value.

(a) Imagine that aggregate demand is boosted by £1million. Find the resulting first-round increase in income, second round increase, third round increase, and so on. When the increases become negligible, add them all together in order to obtain the total increase in income.

(b) Work out the multiplier directly from the *mpc*, and use this to find the income increase that results from the £1million boost to aggregate demand. Check that your answer is the same as in (a). If you have a different answer to (a), why is this?

(c) Use graph paper to confirm the answers you have obtained in (a) and (b). Assume that the aggregate demand equation is:

$$AD = 2 + cY$$

where *c* is your *mpc*. Draw this AD curve on a graph and find the equilibrium income level by locating where it crosses the 45^0 line. Now draw a new AD curve which is the original AD curve shifted upwards by one unit. Find the new equilibrium. How much has income increased? Your answer should be the same as in (a) and (b).

ANSWERS

(a) and (b)

The multiplier is the accurate calculation because it calculates the total change in income by taking account of all successive stages.

MPC	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8	Step 9	Step 10	TOTAL (m)
0.5	£1.00	£0.50	£0.25	£0.13	£0.06	£0.03	£0.02	£0.01	£0.00	£0.00	£2.00
0.55	£1.00	£0.55	£0.30	£0.17	£0.09	£0.05	£0.03	£0.02	£0.01	£0.00	£2.22
0.6	£1.00	£0.60	£0.36	£0.22	£0.13	£0.08	£0.05	£0.03	£0.02	£0.01	£2.48
0.66	£1.00	£0.66	£0.44	£0.29	£0.19	£0.13	£0.08	£0.05	£0.04	£0.02	£2.90
0.7	£1.00	£0.70	£0.49	£0.34	£0.24	£0.17	£0.12	£0.08	£0.06	£0.04	£3.24
0.75	£1.00	£0.75	£0.56	£0.42	£0.32	£0.24	£0.18	£0.13	£0.10	£0.08	£3.77
0.8	£1.00	£0.80	£0.64	£0.51	£0.41	£0.33	£0.26	£0.21	£0.17	£0.13	£4.46
0.85	£1.00	£0.85	£0.72	£0.61	£0.52	£0.44	£0.38	£0.32	£0.27	£0.23	£5.35
0.9	£1.00	£0.90	£0.81	£0.73	£0.66	£0.59	£0.53	£0.48	£0.43	£0.39	£6.51
0.95	£1.00	£0.95	£0.90	£0.86	£0.81	£0.77	£0.74	£0.70	£0.66	£0.63	£8.03

Using multiplier

MPC	Multiplier	Change in Income £m
0.5	2	£2.00
0.55	2.2222	£2.22
0.6	2.5	£2.50
0.66	2.9412	£2.94
0.7	3.3333	£3.33
0.75	4	£4.00
0.8	5	£5.00
0.85	6.6667	£6.67
0.9	10	£10.00
0.95	20	£20.00

6. Top Tips

Students can get a very good 'feel' for how the national income model works through the use of "What..If" scenarios in Excel. Provide students with opportunities to predict the outcome of changing macroeconomic variables in a spreadsheet e.g. the effect on Y of changing the MPC or I etc.

7. Conclusion

For many students the macroeconomic applications will be quite hard. This material is probably best tackled once the microeconomic examples have been thoroughly practised. Higher ability students could be directed towards more detailed and advanced resources which look at further mathematical expressions embedded in the circular flow of income model e.g. taxation, the marginal propensity to import and export, the accelerator theory etc.

© Authored and Produced by the METAL Project Consortium 2007