Indefinite Integrals - ACTIVITIES

Task One

<u>Learning Objectives</u>

LO1: To understand the link between a differentiated function and an integrated function

LO2: To be able to confidently and independently integrate simple functions

Students are divided into an even number of groupings typically of 3 or 4 students in each group. Each group has a number of A4 laminated cards comprising of the following:

y	dx	c	x	f(x)
+	-	x	÷	=
0	1	2	3	4
5	6	7	8	9
0	1	2	3	4
5	6	7	8	9

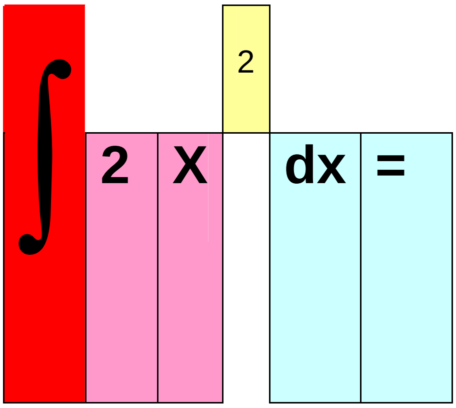
$\int$

The light yellow cards are superscripted numbers and allow powers to be used. A separate and thinner or narrower card is used for the ∫ sign. This allows the figures and variables to be placed next to the integration sign.

One group – the “question setters” - then agrees a function and sets this out using the laminated cards. The other group - the “responders” then has 60 seconds to discuss and agree what the integral of this function is and to then set this out using the laminated cards.

For example:

Group 1: Question Setters



Group 2: Responders

The group would then use their set of cards to, hopefully, create the correct answer i.e. $\frac{2}{3}x^3 + c$

This activity offers numerous benefits. It helps students to understand the structure of the integral – that is, the integral sign, then the function then the “dx” element – and this is reinforced by the use of coloured cards. The activity also offers an active learning approach to integration and this could be extended by asking students to move between different groups and creating more kinaesthetic learning. And, of course, there are also an infinite number of questions and answers which students can create themselves.

Task Two

LO1: To be able to confidently and independently integrate simple functions

LO2: To be able to explain why a particular answer is correct.

Part A

Students are given a prepared sheet of indefinite integration problems and asked to map questions with answers. This activity can be particularly useful for students who may not be the most confident at calculus: in effect, they are given the mapping of answers and they are therefore implicitly guided or supported to find the correct solution.

A typical set of questions and answers is set out below together with the answers:

Integration problem	Answer
$\int 4x^5 dx$	$\frac{x^3}{3} + c$
$\int x^2 dx$	$\frac{-3}{x^2} + c$
$\int \frac{6}{x^3} dx$	$\frac{2x^6}{3} + c$
$\int 12x \quad dx$	$\frac{x^3}{3} + \frac{x^2}{2} + c$
$\int x(x+1)dx)$	$\frac{x^3}{3} + \frac{1}{x} + c$
$\int \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)dx$	$6x^2 + c$

Part B

The Part B tasks are the more ‘traditional’ indefinite integration problems. Higher ability students could think of ways this could be calculated using an Excel spreadsheet and the website http://www.webmath.com/integrate.html might help students to think about how they could produce something similar.

Find:
$\int x^2\,dx$
$\int x^5\,dx$
$\int \left(\frac{1}{x^2}\right) dx$
$\int a^{-7}\,da$
$\int 7 \quad dx$
$\int \left(\frac{5x^2}{\sqrt{x}}\right) dx$
$\int 2x^2(3-4x)\,dx$

Indefinite Integrals - ANSWERS

Part A

Integration problem	Answer
$\int 4x^5 dx$	$\frac{x^3}{3} + c$
$\int x^2 dx$	$\frac{-3}{x^2} + c$
$\int \frac{6}{x^3} dx$	$\frac{2x^6}{3} + c$
$\int 12x \quad dx$	$\frac{x^3}{3} + \frac{x^2}{2} + c$
$\int x(x+1)dx)$	$\frac{x^3}{3} + \frac{1}{x} + c$
$\int \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) dx$	$6x^2 + c$

Part B

Find:	ANSWER
$\int x^2 \quad dx$	$\frac{1}{3}x^3 + c$
$\int x^5 dx$	$\frac{1}{6}x^6 + c$
$\int \left(\frac{1}{x^2}\right) dx$	$-\frac{1}{x}$ **or** $-x^{-1}$
$\int a^{-7} da$	$-\frac{a^{-6}}{6}$ **or** $\frac{-1}{6a^6}$
$\int 7 \quad dx$	7x + c

Find:	ANSWER
$\int\left(\frac{5x^2}{\sqrt{x}}\right)dx$	$2x^{\frac{5}{2}}+c$
$\int 2x^2(3-4x)dx$	$2x^3-2x^4+c$

Definite Integrals - ACTIVITIES

ACTIVITY ONE

Learning Objectives

LO1: Students to learn how to solve simple definite integrals

LO2: Students to consolidate understanding on integrating roots

LO3: Students to learn how to use integration to calculate the area under a graph

Task One

(i) Graph the following functions

(a) $y=3x^2$ for the range -5 to +5

(b) $y=2x$ for the range -5 to +5

(c) $y=\frac{1}{x^2}$ for the range -5 to +5

(ii) Draw the following limits on your graphs

(a) 0 to +2

(b) +1 to +5

(c) +1 to +2

Task Two

Evaluate the following definite integrals:

(a) $\int_0^2 3x^2 \, dx$

(b) $\int_1^5 2x \, dx$

(c) $\int_1^2 \frac{1}{x^2} \, dx$

ACTIVITY TWO

Learning Objectives

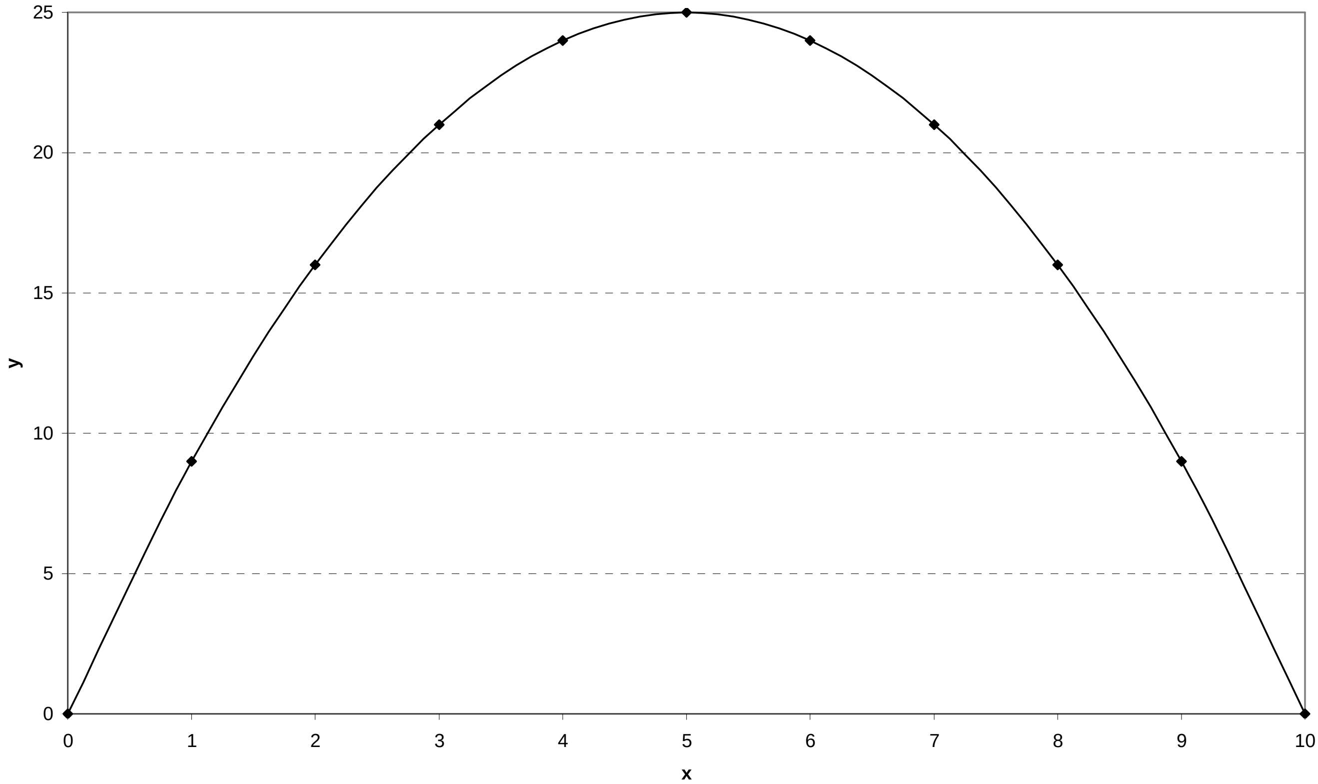
LO1: Students to learn how to calculate the area under a curve

LO2: Students to learn a practical application of integration to microeconomics

Task One

An economist undertakes some simple research on the relationship between profit (y) and quantity sold (x). He determines that:

Y= 10x - x^2 and Y and x have only positive values. The function is shown below



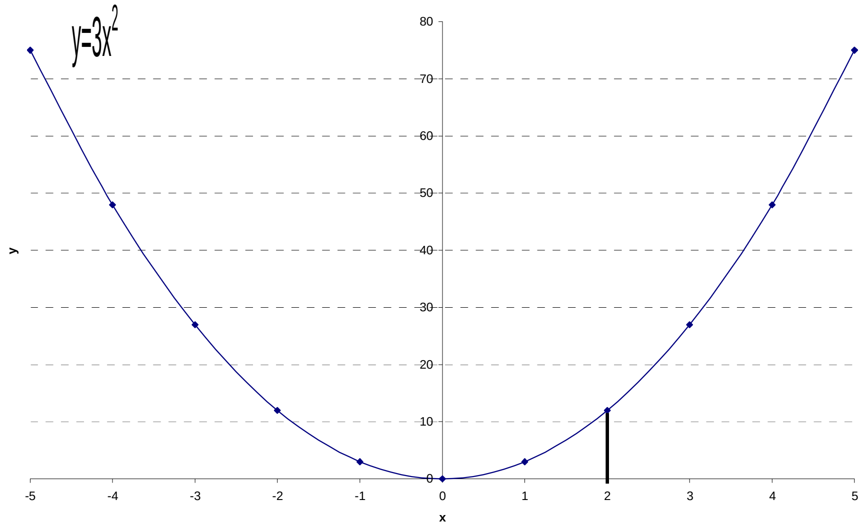
Calculate the area under this function between x=1 and x=10

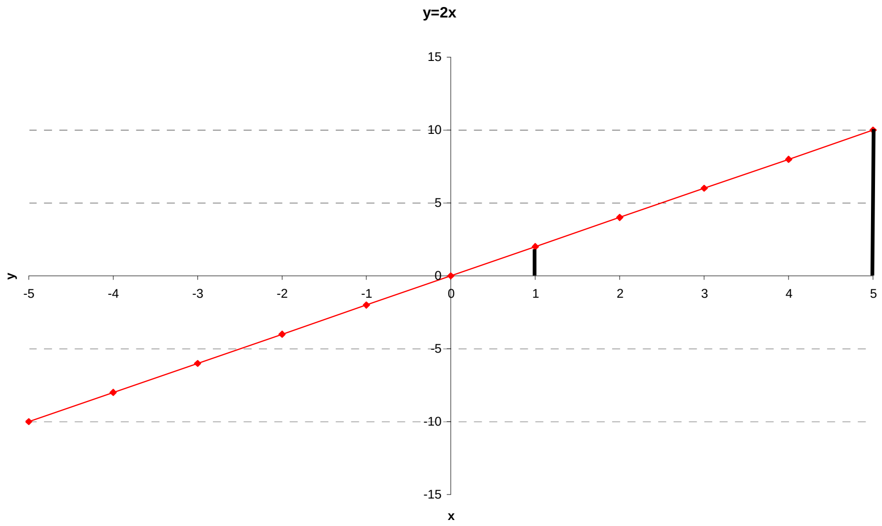
Definite Integrals - ANSWERS

ACTIVITY ONE

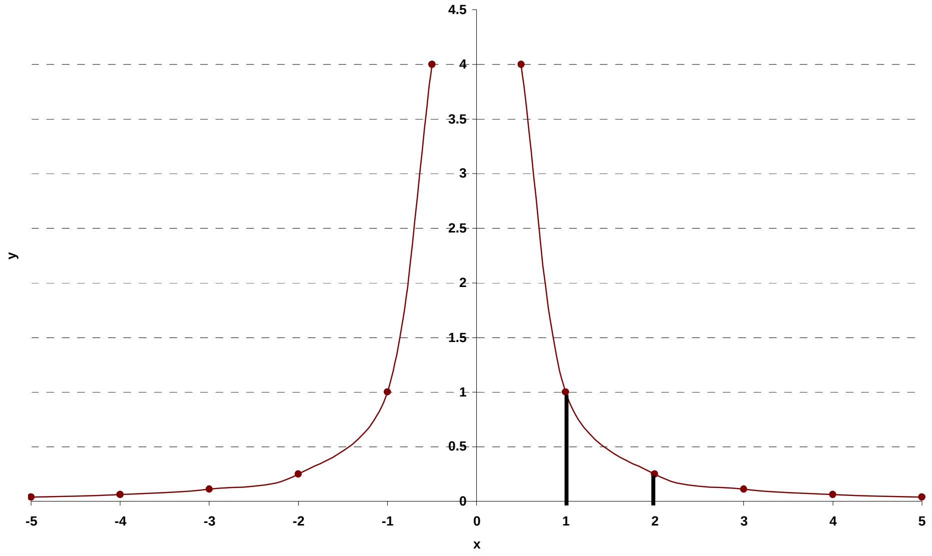
Task One

(i) and (ii)





y= $\frac{1}{x^2}$



Task Two

(a) 8

(b) 24

(c) 0.5

ACTIVITY TWO

Task One

Area = 162

Integration and Exponential Functions - ACTIVITIES

Learning Objectives

LO1: Students to practice integration of exponential functions

LO2: Students to consolidate earlier learning of expanding expressions

Task One

Evaluate:

(a) $\int_{2}^{3} e^{x} \, dx$

(b) $\int_{0}^{1} (3 + e^{x})(2 + e^{-x}) \, dx$

(c) $\int_{0}^{3} e^{-x} \, dx$

Integration and Exponential Functions - ANSWERS

Task One

(a) 12.696

(b) 12.33

(c) 0.95