Graphs and co-ordinates - Activities

ACTIVITY ONE

Learning Objectives

LO1. Students to consolidate meaning of graphs and co-ordinates

LO2. Students to practice plotting simple points

Task One

Draw a graph with each axis having range -5 to +5. The following sets of coordinates, when connected by straight lines in the order specified, each trace out one of the following objects: a square; a rectangle; a diamond; a triangle. Which set of coordinates traces out which object?

A: $(-2,1) \rightarrow (0,-1) \rightarrow (2,1) \rightarrow (0,3) \rightarrow (-2,1)$

B: $(3,0) \rightarrow (4,2) \rightarrow (3,4) \rightarrow (2,2) \rightarrow (3,0)$

C: $(0,-2) \rightarrow (3,1) \rightarrow (4,-2) \rightarrow (2,-2) \rightarrow (0,-2)$

D: $(1,-2) \rightarrow (-2,4) \rightarrow (0,5) \rightarrow (3,-1) \rightarrow (1,-2)$

ACTIVITY TWO

LO1. Students to understand the practical significance of choosing which axis to use for plotting variables

Consider the following pairs of variables. If you were going to plot one against the other, on which axis would you place them? First of all decide (if possible) which variable is independent, and which dependent.

(i) Annual Income of an individual and total Annual Expenditure by the individual.

(ii) Weekly income of an individual and number of hours worked per week by the individual.

(iii) Annual Income of an individual and number of years of education of the individual.

(iv) Price of fuel and quantity consumed of fuel by UK households.

(v) Price of fuel and quantity of fuel supplied to world market by oil-producing countries.

(vi) Number of students attending UK Universities and average tuition fees

Graphs and co-ordinates - ANSWERS
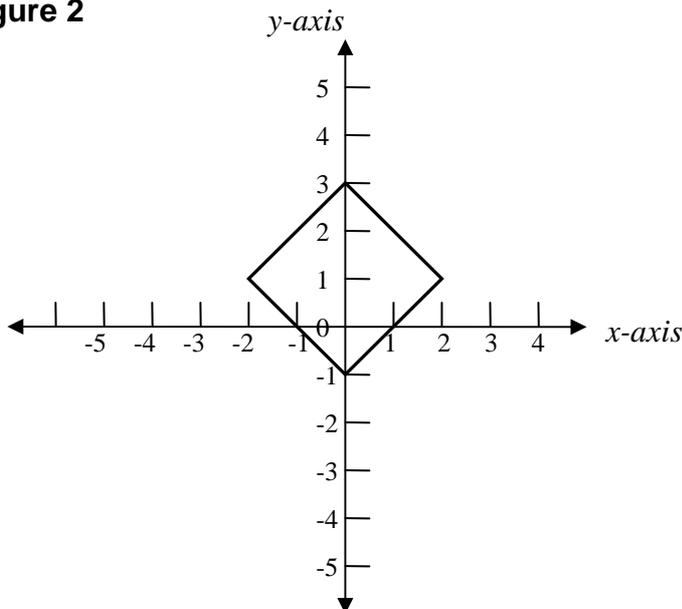
ACTIVITY ONE

A: Square (and illustrated below in **Figure 2**)

B: Diamond

C: Triangle

D: Rectangle

Figure 2

ACTIVITY TWO

(i) Annual Income of an individual **(x)** and total Annual Expenditure by the individual **(y)**

(ii) Weekly income of an individual **(y)** and number of hours worked per week by the individual **(x)**

(iii) Annual Income of an individual **(y)** and number of years of education of the individual **(x).**

(iv) Price of fuel **(y)** and quantity consumed of fuel by UK households **(x).**

(v) Price of fuel and **(y)** quantity of fuel supplied to world market by oil-producing countries **(x)**

(vi) Number of students attending UK Universities **(y)** and average tuition fees **(x)**

Finding the area of a region of a graph - ACTIVITIES

ACTIVITY ONE

LO1: Students to independently construct regions of a graph

LO2: Students to independently calculate the area of a region of a graph

TASK ONE

(a) Plot the following sets of coordinates:

$(-2,1) \rightarrow (0,-1) \rightarrow (2,1) \rightarrow (0,3) \rightarrow (-2,1)$

(b) Confirm that when these points are connected, a perfect square is traced out.

(c) You will see that since the sides of the square are not parallel with the axes, it is not immediately obvious what the length of a side is. One way to compute the area of this square is to consider the larger square shown in Figure 3, given by:

$(-2,-1) \rightarrow (2,-1) \rightarrow (2,3) \rightarrow (-2,3) \rightarrow (-2,-1)$

Figure 3
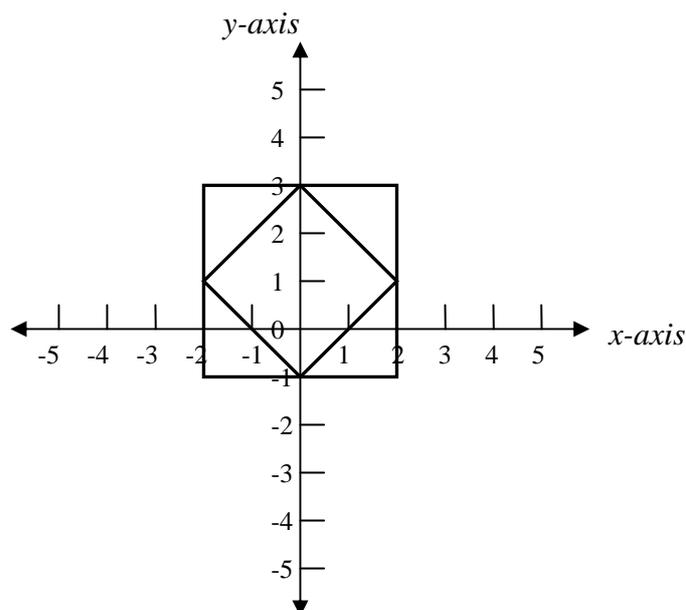
This larger square has sides of length 4 and therefore has area 16 (i.e. 4^2). The area of the smaller square can be deduced by subtracting from 16 the areas of the four triangles outside the smaller square but inside the larger:

Confirm that the area of the smaller square can be calculated by:

$$Area = 16 - \left(4 \times \frac{1}{2} \times 2 \times 2 \right) = 16 - 8 = \underline{\underline{8}}$$

(d) Plot these shapes (from Section 2 above) and calculate their areas:

B: $(3,0) \rightarrow (4,2) \rightarrow (3,4) \rightarrow (2,2) \rightarrow (3,0)$

C: $(0,-4) \rightarrow (2,-2) \rightarrow (4,-4) \rightarrow (2,-6) \rightarrow (0,-4)$

D: $(4,0) \rightarrow (4.5,1) \rightarrow (5,0) \rightarrow (4.5,-1.0) \rightarrow (4,0)$

Finding the area of a region of a graph - ANSWERS

Area of A =8

Area of B = 4

Area of C =8

Area of D = 1

Slope (or gradient) of a line on a graph - ACTIVITES

LO1: Students to learn the formula for calculating a gradient of a straight line

LO2: Students to learn how to calculate the gradient of a straight line

TASK ONE

State the formula for measuring the gradient of a straight line

TASK TWO

   (a) Graph the following two points on a graph: (1,2); (2,4).

   (b) What is the slope of this straight line?

   (c) Interpret the meaning and significance of the "+" sign in front of the gradient figure.

TASK THREE

Choose one of the following pairs of points. For the pair of points you have chosen, carry out the following tasks:

(1)   Plot the two points on a graph.(2) Draw a straight line passing through the two points

(3)   Find the slope of the line; is the line upward-sloping, downward-sloping, or neither?

(4)   Report the coordinates of a different point on the same straight line

Pairs of points:

A: (-2, 4) and (2, 0)

B: (-3, -1) and (4, -1)
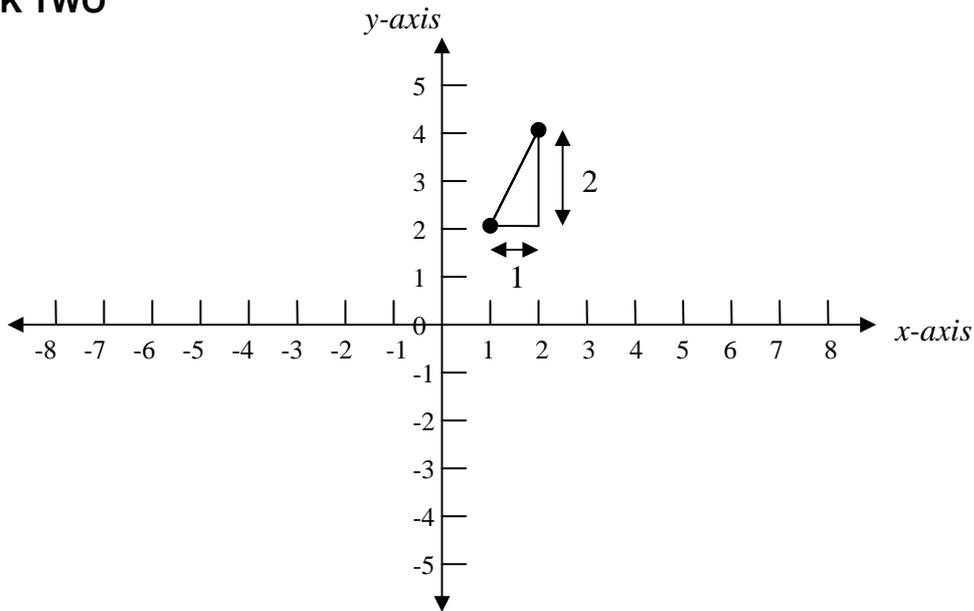
C: (1, 4) and (1, -3)

D: (-1, -2) and (5, 1)

Slope (or gradient) of a line on a graph - ANSWERS

TASK ONE

$$\frac{y_2 - y_1}{x_2 - x_1}$$

TASK TWO

(a)

TASK THREE

Answers depend on the points chosen.

The equation of a straight line: intercept and slope - ACTIVITIES

TASK ONE

LO1: Students to learn to interpret straight line equations

LO2: Students to apply their knowledge and understanding of a straight-line equation to simple Engel curves.

The following equations each show the relationship between household income per week (x) and household expenditure on a particular good per week (y):

(1) $y = 100 + 0.3x$

(2) $y = 200 - 0.15x$

(3) $y = 25$

(4) $y = -200 + 0.05x$

Answer the following questions relating to equations (1)-(4).

(a)   What are these equations called by Economists?

(b)   To what sort of good do you think that each equation relates? Explain your answers.

(c)   In each case, calculate weekly expenditure on the good for a household earning 1000 per week.

(d)   In each case, where applicable, find the income level at which expenditure equals zero.

TASK TWO

There are two Pizzerias in your street. *Mama Mia's* charges £6 for the base and £1 for each topping. *Pepe's Pizzas* charges £3 for the base and £2 for each topping. You have heard that they both supply pizzas of similar size and quality. Let x be the number toppings you want and let y be the amount you pay for the pizza.

(a) Draw a graph with a horizontal scale of 0 to 6 and construct a line for each Pizzeria showing price against number of toppings. Label the two lines.

(b) Using the graph, consider how the number of toppings would determine your choice of Pizzeria. Is there any situation in which you would be indifferent?

TASK THREE

LO1: Students to learn how to independently write simple straight line equations

LO2: Students to learn how make inferences using a straight line.

LO3: Students to learn how to invert a straight line equation.

It is well known that to convert temperature in degrees Centigrade to temperature in degrees Fahrenheit, you do the following, in the specified order:

Multiply by 9; divide by 5; add 32.

Let x be temperature in degrees Centigrade, and let y be temperature in degrees Fahrenheit.

(a) According to the above rule, write down an equation which shows y as a function of x. Your equation should be the equation of a straight line. What is the intercept, and what is the slope?

(b) Sketch the line on a graph, with x measured on the horizontal axis, and y on the vertical.

(c) Use your equation to answer the following:

(i) Water freezes at zero degrees Centigrade. At what temperature does water freeze in degrees Fahrenheit?

(ii) Water boils at 100 degrees Centigrade. At what temperature does water boil in degrees Fahrenheit?

(iii) "Absolute Zero" is the lowest temperature that it is theoretically possible to attain. It is -273 degrees Centigrade. What is Absolute Zero in degrees Fahrenheit?

(d)

(i) Next, <u>invert</u> your equation, so that it shows x as a function of y.

(ii) Sketch the straight line represented by the new equation on a graph. Note that for this graph, y should be measured on the horizontal, and x on the vertical.

(iii) Use this second equation to answer the following. "Normal" body temperature is 98.6 degrees Fahrenheit. What is normal body temperature in degrees Centigrade?

The equation of a straight line: intercept and slope - ANSWERS

TASK ONE

(a) These equations represent Engel curves: relationships between the quantity demanded of a good or service and levels of income.

(b) Goods (1) and (4) are normal goods. Good (2) is an inferior good. Good (3) posits there is no relationship between the quantity demanded and income level.

(c) Good 1: $y = 400$, Good 2: $y = 50$, Good 3: $y = 25$, Good 4: $y = -150$

(d) Good 1: x= -£333.33, Good 2: x= £1333.33, Good 3: x =£0, Good 4: x=£4000

TASK TWO

   (a) see below

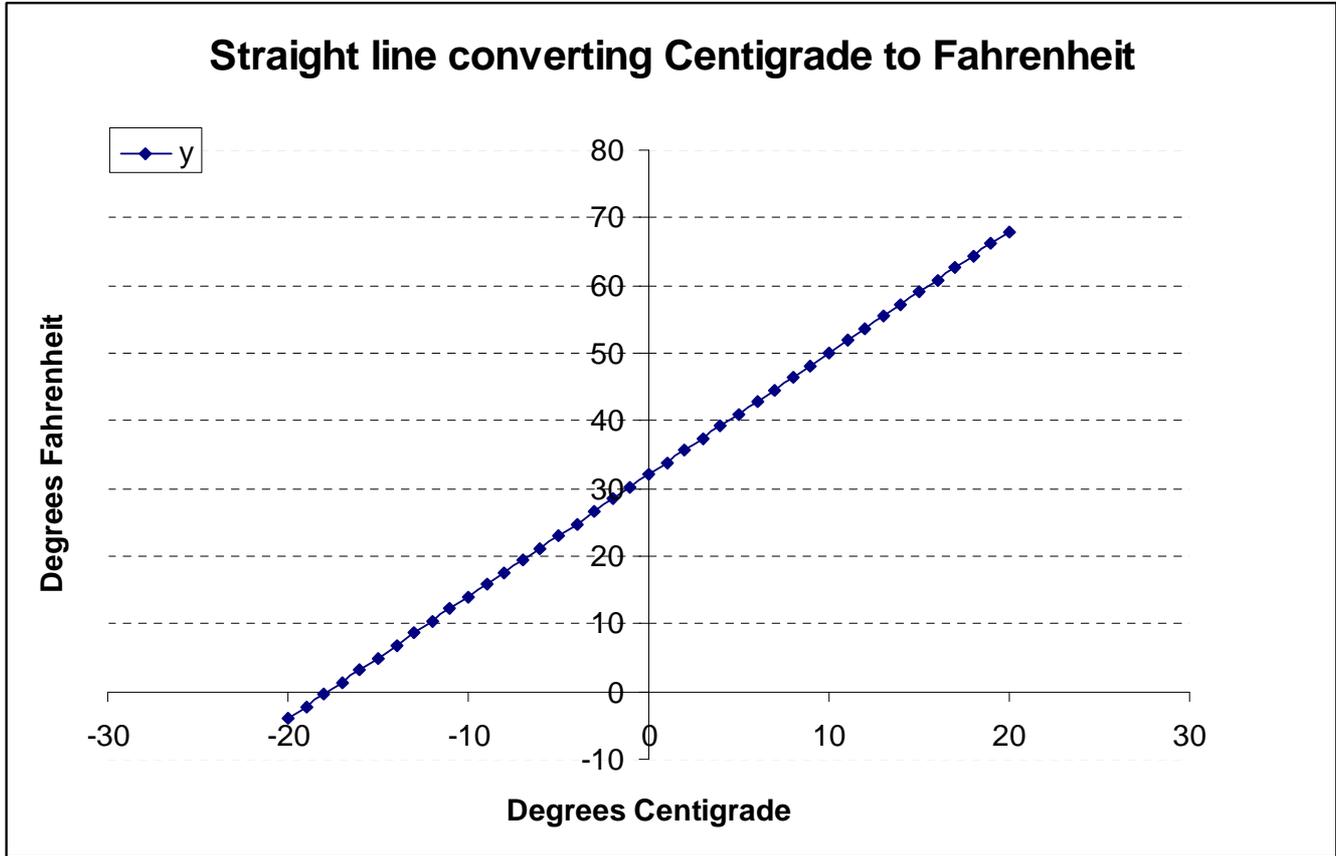**Equations to show the costs of pizzas**

   (b) At 3 toppings the cost at either pizzeria is £9 and assuming that quality is identical the consumer would be indifferent.

TASK THREE

(a) $y = 32 + \dfrac{9}{5}x$

(b)

Straight line converting Centigrade to Fahrenheit

(c)

(i) 32^0F

(ii) 212^0F

(iii) -459.4^0F

(d)

(i) The equation for converting temperatures from degrees Centigrade (x) to degrees Fahrenheit (y) is:

$$y = 32 + \frac{9}{5}x$$

To invert this equation, we do the following:

$$-32 + y = \frac{9}{5}x$$

$$\therefore \frac{9}{5}x = -32 + y$$

$$\therefore x = -32\left(\frac{5}{9}\right) + \left(\frac{5}{9}\right)y$$

$$\therefore \underline{x = -17.778 + 0.556\,y}$$

This equation can be used to convert from Fahrenheit (*y*) to Centigrade (*x*).

ii) and (iii) Using the new graph and/or equation, we get 37^{0}C.